



National Aeronautics and  
Space Administration

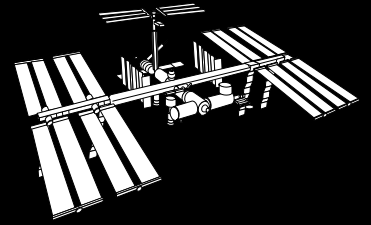
Educational Product

Teachers | Grades 5-12

# Educational Brief

**Subject:** Mathematics

**Topic:** Microgravity

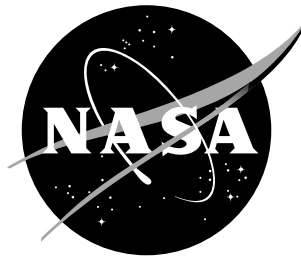


# THE MATHEMATICS OF MICROGRAVITY

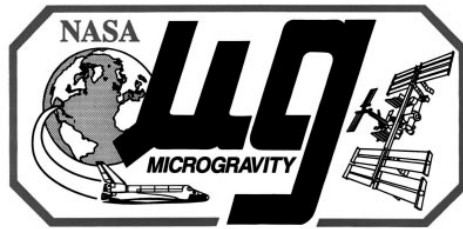


$$F = G \frac{m_1 m_2}{r^2}$$

# The Mathematics of Microgravity



**National Aeronautics and Space Administration**



**Office of Life and Microgravity Sciences and Applications  
Microgravity Science and Applications Division**

**Office of Human Resources and Education  
Education Division**

This publication is in the Public Domain and is not protected by copyright.  
Permission is not required for duplication.

EB-1997-02-119-HQ

---

## Acknowledgments

This publication was developed for the National Aeronautics and Space Administration.

Writers:

**Melissa J. B. Rogers, MS**  
TAL-CUT Company  
NASA Lewis Research Center  
Cleveland, OH

**Gregory L. Vogt, Ed.D.**  
Teaching From Space Program  
NASA Johnson Space Center  
Houston, TX

**Michael J. Wargo, Sc.D.**  
Microgravity Science and Applications  
Division  
NASA Headquarters  
Washington, DC

Editor:

**Carla B. Rosenberg**  
Teaching From Space Program  
NASA Headquarters  
Washington, DC

### How To Use This Educational Brief

The main text of the educational brief uses large print and is located in the wide column. Subjects relating to mathematics and physics principles are highlighted in bold. Mathematics standards, questions for discussion, examples, and derivations are provided in smaller print in the narrow column of each page.

Each area highlighted in the text has a corresponding section in the narrow column. This corresponding section first lists the applicable **Mathematics Standards**, indicated by grade level. We have attempted to position the appropriate mathematics discussion as close as possible to the relevant highlighted text.

# Introduction

## Mathematics Standards

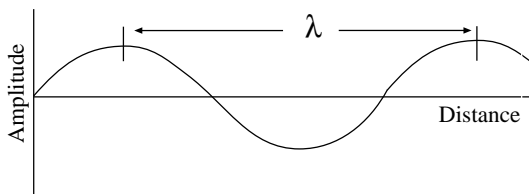
Grades 5-8 ( $\Delta$ ); Grades 9-12 ( $\square$ )

- Mathematics as Communication
- Mathematical Connections
- $\Delta$   Number and Number Relationships
- $\Delta$  Number Systems and Number Theory

The electromagnetic spectrum is generally separated into different radiation categories defined by frequency (units of Hertz) or wavelength (units of meters). Wavelength is commonly represented by the symbol  $\lambda$ .

Examples:

Name	Approximate Wavelength (m)
X-rays	$= 10^{-15}$ to $10^{-9}$
Ultraviolet	$= 10^{-8}$ to $10^{-7}$
Visible Light	$= 10^{-7}$ to $10^{-6}$
Infrared	$= 10^{-6}$ to $10^{-3}$
Microwave	$= 10^{-3}$ to $10^{-1}$
Television	$= 10^{-1}$ to 1
AM Radio	$= 10^2$ to $10^3$



## Mathematics Standards

Grades 5-8 ( $\Delta$ ); Grades 9-12 ( $\square$ )

- $\Delta$   Algebra
- $\Delta$   Mathematics as Communication
- $\Delta$  Measurement

$$1 \text{ g} \cong 9.8 \text{ m/s}^2$$
$$\cong 32 \text{ ft/s}^2$$

Units conversion is very important in all areas of life, for example in currency exchange, airplane navigation, and scientific research.

Space flight is important for many reasons. Space flight carries scientific instruments and sometimes humans high above the ground, permitting us to see Earth as a planet and to study the complex interactions of atmosphere, oceans, land, energy, and living things. Space flight lifts scientific instruments above the filtering effects of the atmosphere, making the entire **electromagnetic spectrum** available and allowing us to see more clearly the distant planets, stars, and galaxies. Space flight permits us to travel directly to other worlds to see them up close and sample their compositions. Finally, space flight allows scientists to investigate the fundamental states of matter—solids, liquids, and gases—and the forces that affect them in a microgravity environment.

The study of the states of matter and their interactions in microgravity is an exciting opportunity to expand the frontiers of science. Investigations include biotechnology, combustion science, fluid physics, fundamental physics, and materials science.

Microgravity is the subject of this brief. This publication identifies the underlying mathematics and physics principles that apply to microgravity and acts as a supplement to *Microgravity: Activity Guide for Science, Mathematics, and Technology Education* (EG-1997-02-110-HQ).

## What Is Microgravity?

The presence of Earth creates a gravitational field that acts to attract objects with a force inversely proportional to the square of the distance between the center of the object and the center of Earth. When we measure the acceleration of an object acted upon only by Earth's gravity at Earth's surface, we commonly refer to it as **one g or one Earth gravity**. This acceleration is approximately **9.8 meters per second squared ( $\text{m/s}^2$ )**.



We can interpret the term *microgravity* ( $\mu\text{g}$ ) in a number of ways, depending upon the context. The prefix micro- ( $\mu$ ) derives from the original Greek *mikros*, meaning "small." By this definition, a microgravity environment is one that imparts to an object a net acceleration that is small compared with that produced by Earth at its surface. We can achieve such an environment by using various methods including Earth-based drop towers, parabolic aircraft flights, and Earth-orbiting laboratories. In practice, such accelerations will range from about one percent of Earth's gravitational acceleration (aboard aircraft in parabolic flight) to better than one part in a million (for example, aboard Earth-orbiting free flyers). Earth-based drop towers create microgravity environments with intermediate values of residual acceleration.

Quantitative systems of measurement, such as the metric system, commonly use **micro-** to mean *one part in a million*. By this second definition, the acceleration imparted to an object in microgravity will be **one-millionth ( $10^{-6}$ ) of that measured at Earth's surface**.

The use of the term microgravity in this guide corresponds to the first definition: small gravity levels or low gravity. As we describe how low-acceleration environments can be produced, you will find that the fidelity (quality) of the microgravity environment will depend on the mechanism used to create it. For illustrative purposes only, we will provide a few simple quantitative examples using the second definition. The examples attempt to provide insight into what might be expected if the local acceleration environment were reduced by six orders of magnitude from 1 g to  $10^{-6}$  g.

If you stepped off a roof that was **five meters** high, you would reach the ground in just **one second**. In a microgravity environment equal to **one percent** of Earth's gravitational pull, the same drop would take **10 seconds**. In a microgravity environment equal to **one-millionth** of Earth's gravitational pull, the same drop would take **1,000 seconds**, or about **17 minutes!**

### Mathematics Standards

Grades 5-8 ( $\Delta$ ); Grades 9-12 ( $\square$ )

- $\Delta \square$  Mathematical Connections
- $\Delta \square$  Mathematics as Communication

$$1 \text{ micro-g or } 1 \mu\text{g} = 1 \times 10^{-6} \text{ g}$$

#### Questions for Discussion

- What other common prefixes or abbreviations for powers of ten do you know or can you find?
- In what everyday places do you see these used? *Grocery stores, farms, laboratories, sporting facilities, pharmacies, machine shops.*

Common prefixes for powers of ten:

$10^{-9}$	nano-	n
$10^{-3}$	milli-	m
$10^{-2}$	centi-	c
$10^3$	kilo-	k
$10^6$	mega-	M
$10^9$	giga-	G

### Mathematics Standards

Grades 5-8 ( $\Delta$ ); Grades 9-12 ( $\square$ )

- $\Delta \square$  Algebra
- $\Delta$  Computation and Estimation
  - $\square$  Conceptual Underpinnings of Calculus
  - $\square$  Discrete Mathematics
- $\Delta \square$  Mathematical Connections
- $\Delta \square$  Mathematics as Problem Solving
- $\Delta \square$  Mathematics as Reasoning
- $\Delta$  Number and Number Relationships

Calculate the times in these examples. Teachers can use these examples at several different scholastic levels.

Provide the equation as:

$$t = \sqrt{\frac{2d}{a}} \text{ or } \left(\frac{2d}{a}\right)^{1/2}$$

Provide the equation as  $d=(1/2)at^2$ , and have the students re-order the equation.

Making measurements and calculating results involve the concepts of accuracy and precision, significant figures, and orders of magnitude. With these concepts in mind, are the drop times given in the text "correct"?



## Mathematics Standards

Grades 5-8 (Δ); Grades 9-12 (□)

- Δ □ Algebra
- Δ Computation and Estimation
- Δ □ Mathematical Connections
- Δ □ Mathematics as Problem Solving
- Δ □ Mathematics as Reasoning
- Δ Measurement

### Questions for Discussion

- How far away is the Moon?
- How far away is the center of Earth from the center of the Moon?
- Why did we ask the previous question?
- How far away is the surface of Earth from the surface of the Moon?
- What are the elevations of different features of Earth and the Moon?
- How are elevations measured?

Scientists can create microgravity in two ways. Because gravitational pull diminishes with distance, one way to create a microgravity environment is to travel away from Earth. To reach a point where Earth's gravitational pull is reduced to one-millionth of that at the surface, you would have to travel into space a distance of **6.37 million kilometers from Earth (almost 17 times farther away than the Moon)**.

This approach is impractical, except for automated spacecraft, since humans have yet to travel farther away from Earth than the distance to the Moon. However, the act of free fall can create a more practical microgravity environment.

We will use a simple example to illustrate how free fall can create microgravity (Figure 1). Imagine riding in an elevator to the top floor of a very tall building. At the top, the cables supporting the car break, causing the car and you to fall to the ground. (In this example, we discount the effects of air friction and elevator safety mechanisms on the falling car.) Because you

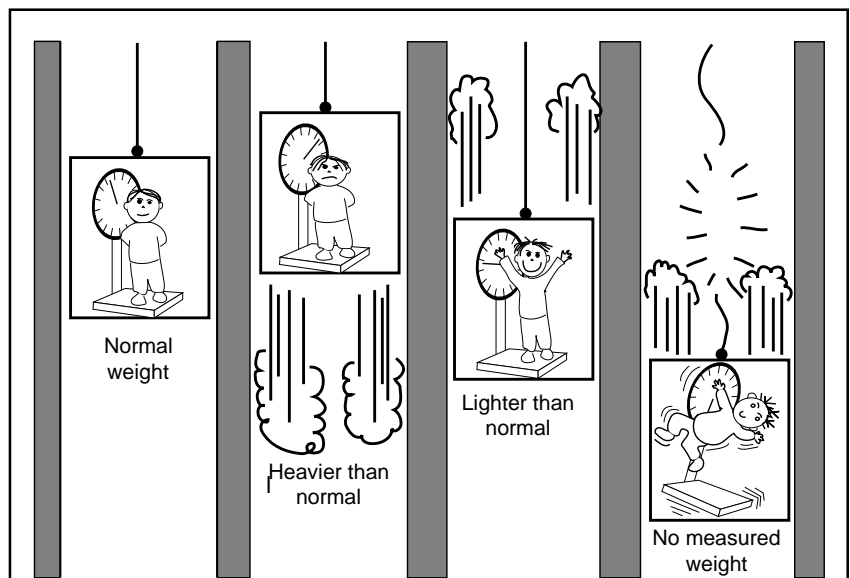


Figure 1. Acceleration and weight

The person in the stationary elevator car experiences normal weight. In the car immediately to the right, weight increases slightly because of the upward acceleration. Weight decreases slightly in the next car because of the downward acceleration. A scale does not measure weight in the last car on the right because of free fall.



and the elevator car are falling together, acted on only by gravity, you will float inside the car if you lift your feet off the elevator floor. In other words, you and the elevator car accelerate downward at the same rate, which is due to gravity alone. **If a scale were present, your weight would not register because the scale would be falling too.**

## Gravity

Gravitational attraction is a fundamental property of matter that exists throughout the known universe. Physicists identify gravity as one of the four types of forces in the universe. The others are the strong and weak nuclear forces and the electromagnetic force.

More than 300 years ago the great English scientist Sir Isaac Newton published the important generalization that mathematically describes this universal force of gravity. Newton was the first to realize that gravity extends well beyond the domain of Earth. The basis of this realization stems from the first of three laws he formulated to describe the motion of objects. Part of Newton's first law, the law of inertia, states that objects in motion travel in a straight line at a constant velocity unless acted upon by a net force. According to this law, the planets in space should travel in straight lines. However, as early as the time of Aristotle, scholars knew that the planets travelled on curved paths. Newton reasoned that the circular motions of the planets are the result of a net force acting upon each of them. That force, he concluded, is the same force that causes an apple to fall to the ground—gravity.

Newton's discovery of the universal nature of the force of gravity was remarkable. To take the familiar force that makes an apple fall to Earth and be able to recognize it as the same force that keeps the planets on their quiet and predictable paths represents one of the major achievements of human intellectual endeavor. This ability to see beyond the obvious and familiar is the mark of a true visionary. Sir Isaac Newton's pioneering work epitomizes this quality.

### Mathematics Standards

Grades 5-8 (Δ); Grades 9-12 (□)

- Δ □ Mathematical Connections
- Δ □ Mathematics as Reasoning

#### Questions for Discussion

- How does a scale work?
- What does a scale measure?
- How many different kinds of scales can you list?
- Do they need gravity for them to work?
- Would you get different results on the Moon or Mars?
- How can you measure the mass of an object in microgravity? Hint: See Inertial Balance Activity in the *Microgravity Activities Guide*, EG-1997-02-110-HQ.

### Mathematics Standards

Grades 5-8 (Δ); Grades 9-12 (□)

- Δ □ Algebra
- Δ Computation and Estimation
  - Functions
- Δ □ Mathematics as Communication
- Δ Number and Number Relationships
- Δ Patterns and Functions

Introduction of concepts of proportionality versus equality and mathematical expressions versus equations.

$$F \propto \frac{m_1 m_2}{r^2} \quad \text{indicates proportionality}$$

$$F = G \frac{m_1 m_2}{r^2} \quad \text{indicates equality}$$

$$G \frac{m_1 m_2}{r^2} \quad \text{is an expression}$$

$$F = G \frac{m_1 m_2}{r^2} \quad \text{is an equation}$$



### Mathematics Standards

Grades 5-8 (Δ); Grades 9-12 (◻)

- Δ ◻ Algebra
- Δ ◻ Mathematical Connections
- Δ ◻ Mathematics as Communication
- Δ Measurement

The internationally recognized Systeme International (SI) is a system of measurement units. The SI units for length (meter) and mass (kg) are taken from the metric system. Many dictionaries and mathematics and science textbooks provide conversion tables between the metric system and other systems of measurement.

#### Units Conversion Examples

- 1 kg ≅ 2.2 lb      1 in ≅ 2.54 cm
- 1 liter ≅ 1 qt      1 yd ≅ 0.9 m

#### Questions for Discussion

- What common objects have a mass of about 1 kg?
- What are the dimensions of this sheet of paper in cm and inches?
- How many liters are there in a gallon?

### Mathematics Standards

Grades 5-8 (Δ); Grades 9-12 (◻)

- Δ Computation and Estimation
- Δ ◻ Mathematics as Communication
- Δ Number and Number Relationships
- Δ Number Systems and Number Theory

Scientific notation makes it easier to read, write, and manipulate numbers with many digits. This is especially useful for making quick estimates and for indicating the number of significant figures.

#### Examples:

$$\begin{aligned} 0.001 &= 10^{-3} \\ 10 &= 10^1 \\ 1000 &= 10^3 \end{aligned}$$

Which is bigger,  $6 \times 10^{-3}$  or  $8 \times 10^{-4}$ ?  $6 \times 10^3$  or  $8 \times 10^4$ ? How much bigger?

Newton's experimental research into the force of gravity resulted in his elegant mathematical statement that is known today as the *Law of Universal Gravitation*. According to Newton, every mass in the universe attracts every other mass. The attractive force between any two objects is **directly proportional to the product of the two masses being measured and inversely proportional to the square of the distance separating them**. If we let  $F$  represent this force,  $r$  represent the distance between the centers of the masses, and  $m_1$  and  $m_2$  represent the magnitude of the masses, the relationship stated can be written symbolically as:

$$F \propto \frac{m_1 m_2}{r^2}$$

(We define  $\propto$  mathematically to mean "is proportional to.") From this relationship, we can see that the greater the masses of the attracting objects, the greater the force of attraction between them. We can also see that the farther apart the objects are from each other, the less the attraction. Note that the inverse square relationship with respect to distance is important. In other words, if the distance between the objects doubles, the attraction between them diminishes by a factor of four, and if the distance triples, the attraction is only one-ninth as much.

The eighteenth-century English physicist Henry Cavendish later quantified Newton's Law of Universal Gravitation. He actually measured the gravitational force between two **one kilogram masses** separated by a distance of one meter. This attraction was an extremely weak force, but its determination permitted the proportional relationship of Newton's law to be converted into an equation. This measurement yielded the universal gravitational constant or  $G$ . Cavendish determined that **the value of  $G$  is  $0.0000000000667$  newton  $m^2/kg^2$  or  $6.67 \times 10^{-11} Nm^2/kg^2$** . With  $G$  added to the equation, the Universal Law of Gravitation becomes:

$$F = G \frac{m_1 m_2}{r^2}$$





## Deep In Space

You can use the **inverse square relationship**, with respect to distance, of the Law of Gravitation to determine how far to move a microgravity laboratory from Earth to achieve a  $10^{-6}$  g environment. Distance ( $r$ ) is measured between the centers of mass of the laboratory and of Earth. While the laboratory is still on Earth, the distance between their centers is 6,370 km (equal to the approximate radius of Earth,  $r_e$ ). To achieve  $10^{-6}$  g, the laboratory has to be moved to a distance of 1,000 Earth radii. In the equation,  $r$  then becomes  $1,000 r_e$  or  $r = 6.37 \times 10^6$  km.

## Creating Microgravity

### Drop Towers and Tubes

In a practical sense, we can achieve microgravity with a number of technologies, each depending upon the act of free fall. Drop towers and drop tubes are high-tech versions of the elevator analogy presented in a previous section. (The large version of these facilities is essentially a hole in the ground.)

NASA's Lewis Research Center in Cleveland, Ohio has a 145 meter drop tower facility that begins on the surface and descends into Earth like a mine shaft. The test section of the facility is 6.1 meters in diameter and 132 meters deep. Beneath the test section is a catch basin filled with polystyrene beads. The 132 meter drop creates a microgravity environment for a period of 5.2 seconds.

To begin a drop experiment, one places the experiment apparatus in a cylindrical test vehicle that can carry **experiment loads of up to 450 kilograms**. The vehicle hangs suspended from a cap that encloses the upper end of the facility. **Air is pumped out of the facility until a vacuum of**

### Mathematics Standards

Grades 5-8 ( $\Delta$ ); Grades 9-12 ( $\square$ )

- $\Delta \square$  Algebra
- $\Delta$  Computation and Estimation
  - $\square$  Conceptual Underpinnings of Calculus
  - $\square$  Discrete Mathematics
- $\Delta \square$  Mathematical Connections
- $\Delta \square$  Mathematics as Problem Solving
- $\Delta \square$  Mathematics as Reasoning
- $\Delta$  Number and Number Relationships

The inverse square relationship can also be used to determine the acceleration due to gravity at any distance from the center of Earth,  $r$ .

$$F = \frac{Gm_e m}{r_e^2} = \begin{array}{l} \text{Force of gravity on mass,} \\ \text{m, at Earth's surface} \end{array}$$
$$= mg \quad \text{From Newton's second law}$$

$$g = \frac{Gm_e}{r_e^2}$$

$$a = \frac{Gm_e}{r^2} = \begin{array}{l} \text{Acceleration due to} \\ \text{gravity at distance, r} \end{array}$$

$$\frac{a}{g} = \frac{r_e^2}{r^2}$$

$$a = \frac{r_e^2}{r^2} g$$

### Mathematics Standards

Grades 5-8 ( $\Delta$ ); Grades 9-12 ( $\square$ )

- $\Delta \square$  Mathematical Connections
- $\Delta \square$  Mathematics as Reasoning

The mass of an object describes how much the object accelerates under a given force. In British units (commonly used in the United States), force is given in units of pounds. The force due to gravity is called weight. It is incorrect, however, to refer to the mass of a substance in terms of pounds. The British unit of mass corresponding to 1 pound force is the slug.

#### Question for Discussion

- What can you think of that has a mass of about 450 kilograms?
- Would it fit in a 6.1 meter diameter tube?



### Mathematics Standards

Grades 5-8 ( $\Delta$ ); Grades 9-12 ( $\square$ )

- $\Delta \square$  Algebra
- $\Delta$  Computation and Estimation
  - $\square$  Discrete Mathematics
  - $\square$  Functions
- $\Delta \square$  Mathematical Connections
- $\Delta \square$  Mathematics as Problem Solving
- $\Delta$  Number and Number Relationships
- $\Delta$  Patterns and Functions

#### Questions for Discussion

- What percentage of atmospheric pressure is  $10^{-2}$  torr?
- By what percentage is the pressure in the facility reduced?
- Atmospheric pressure differs at different locations on Earth. Investigate how it varies. On what types of things does this variation depend?

### Mathematics Standards

Grades 5-8 ( $\Delta$ ); Grades 9-12 ( $\square$ )

- $\Delta \square$  Mathematical Connections
- $\Delta$  Measurement
- $\Delta \square$  Statistics

Telemetry is the process of transmitting data from a remote measurement location to a receiving station for recording and analysis.

#### Questions for Discussion

- What are different ways you transmit and record data?
- What errors do these methods involve?

**$10^{-2}$  torr is achieved. (Atmospheric pressure is 760 torr.)** Doing so reduces the acceleration effects caused by aerodynamic drag on the vehicle to less than  $10^{-5}$  g. During the drop, cameras within the vehicle record the action and experiment data are **telemetered to recorders.**

The NASA Marshall Space Flight Center in Huntsville, Alabama houses a smaller facility for microgravity research. It is a 100 meter high, 25.4 centimeter diameter evacuated drop tube that can achieve microgravity for periods of as long as 4.5 seconds. A stainless steel bell jar fits in the upper end of the tube. For solidification experiments, scientists mount an electron bombardment or an electromagnetic levitator furnace inside the jar to melt the test samples. After the sample melts, drops form and fall through the tube to a detachable catch fixture at the bottom of the tube (Figure 2).

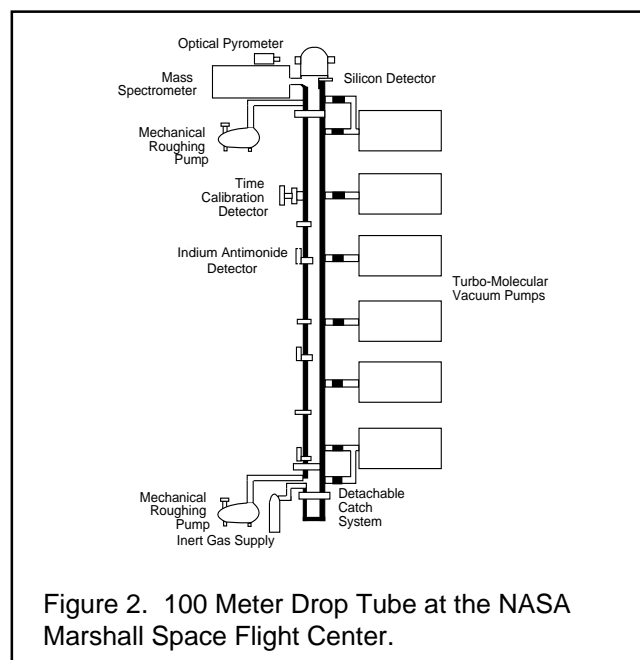


Figure 2. 100 Meter Drop Tube at the NASA Marshall Space Flight Center.



NASA Field Centers and other countries have additional drop facilities of varying sizes to serve different purposes. A 490 meter deep vertical mine shaft in Japan has been converted to a drop facility that can achieve a  $10^{-5}g$  environment for up to 10 seconds.

### Mathematics Standards

Grades 5-8 ( $\Delta$ ); Grades 9-12 ( $\square$ )

- $\Delta \square$  Algebra
- $\Delta$  Computation and Estimation
  - $\square$  Conceptual Underpinnings of Calculus
  - $\square$  Discrete Mathematics
  - $\square$  Functions
- $\Delta \square$  Mathematical Connections
- $\Delta \square$  Mathematics as Problem Solving
- $\Delta \square$  Mathematics as Reasoning
- $\Delta$  Patterns and Functions
- $\Delta \square$  Statistics

### Questions for Discussion

- What is the functional relationship between acceleration, distance, and time?

Use the three sets of drop tower data points given in the text and the additional data set: (24 meters, 2.2 seconds)

This data set is for the drop tower in Figure 3.

How can you get another data set?  
(0 meters, 0 seconds).

*Suggested solution methods: Use different types of graph paper. Use a computer curve fitting program.*

- Knowing that  $g = 9.8 \text{ m/s}^2$ , what equation can you write to incorporate acceleration, distance, and time?
- Assume it costs \$5000 per meter of height to build a drop tower.

How much does it cost to build a drop tower to allow drops of 1 second, 2 seconds, 4 seconds, 10 seconds?

Why does it cost so much more for the longer times?

What would be an inexpensive way to double low-gravity time in a drop tower?

*Shoot the experiment package up from the bottom.*

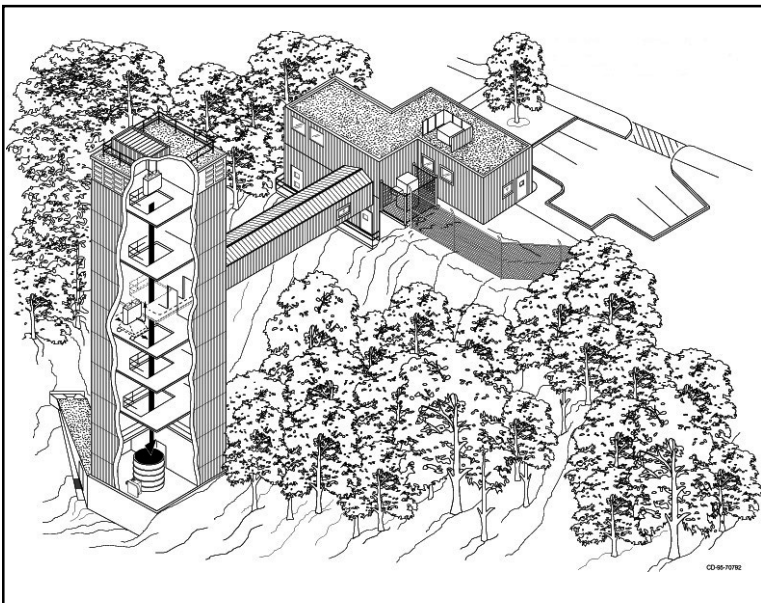


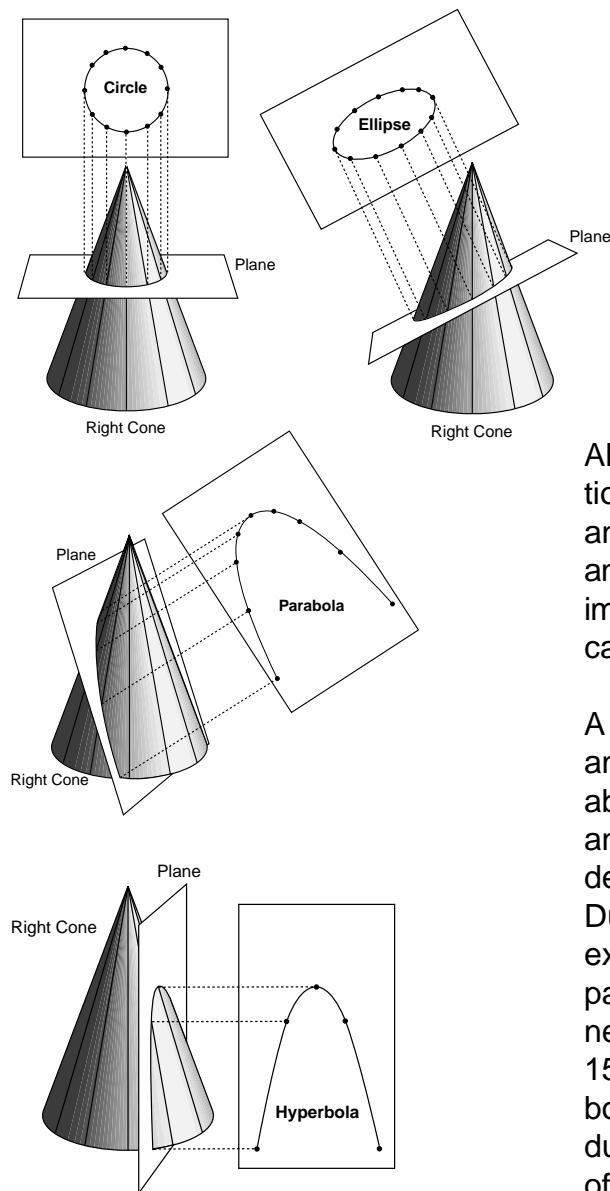
Figure 3. 2.2 Second Drop Tower at the NASA Lewis Research Center.

## Mathematics Standards

Grades 5-8 ( $\Delta$ ); Grades 9-12 ( $\square$ )

- $\square$  Conceptual Underpinnings of Calculus
- $\square$  Functions
- $\Delta \square$  Mathematical Connections
- $\Delta$  Patterns and Functions

Microgravity carriers and other spacecraft follow paths best described by conic sections. The aircraft and sub-orbital rockets trace out parabolas. Orbiting spacecraft are free falling on elliptical paths. When a meteoroid is on a path that is influenced by Earth or any other planetary body but does not get captured by the gravitational field of the body, its motion, as it approaches then moves away from the body, traces out a hyperbolic path.



## Aircraft

Airplanes can achieve low-gravity for periods of about 25 seconds or longer. The NASA Johnson Space Center in Houston, Texas operates a KC-135 aircraft for astronaut training and for conducting experiments. The plane is a commercial-sized transport jet (Boeing 707) with most of its passenger seats removed. Padded walls protect the people inside.

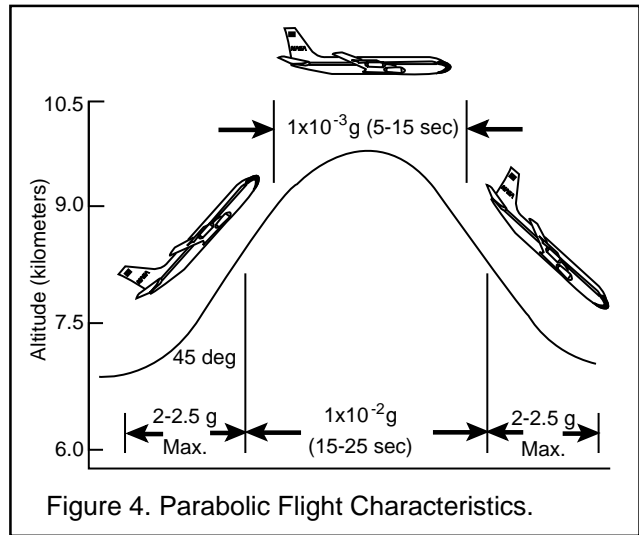


Figure 4. Parabolic Flight Characteristics.

Although airplanes cannot achieve microgravity conditions of as high quality as those produced in drop towers and drop tubes (since they are never completely in free fall and their drag forces are quite high), they do offer an important advantage over drop facilities—experimenters can ride along with their experiments.

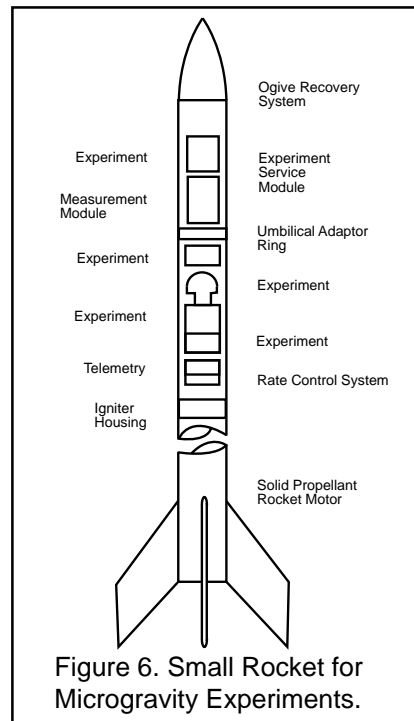
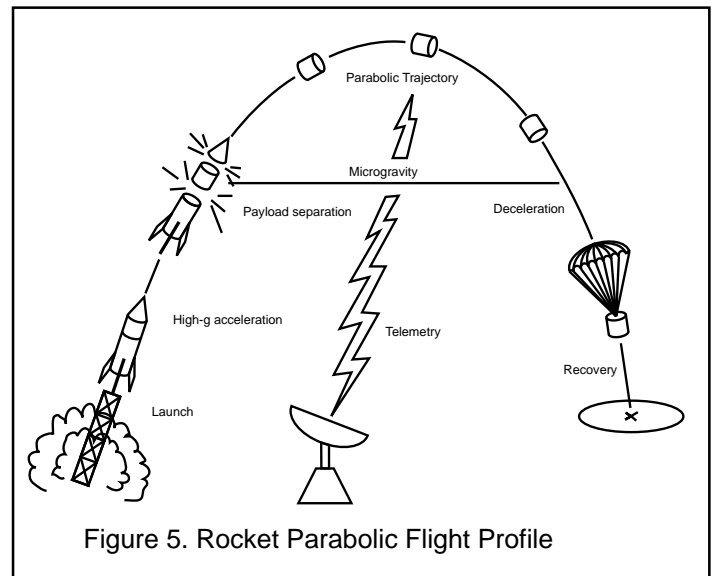
A typical flight lasts 2 to 3 hours and carries experiments and crew members to a beginning altitude about 7 km above sea level. The plane climbs rapidly at a 45 degree angle (pull up), **traces a parabola** (pushover), and then descends at a 45-degree angle (pull out) (Figure 4). During the pull up and pull out segments, crew and experiments experience between 2 g to 2.5 g. During the parabola, at altitudes ranging from 7.3 to 10.4 kilometers, net acceleration drops as low as  $1.5 \times 10^{-2} g$  for more than 15 seconds. On a typical flight the KC-135 flies 40 parabolic trajectories. The gut-wrenching sensations produced on the flight have earned the plane the nickname of the "vomit comet."



NASA also operates a DC-9 for low-gravity research out of the NASA Lewis Research Center. Flying on a trajectory similar to the one followed by the KC-135, the DC-9 provides a low-acceleration environment of  $1.5 \times 10^{-2} \text{ g}$  to  $2 \times 10^{-2} \text{ g}$  for more than 15 seconds.

## Rockets

Small rockets provide a third technology for creating microgravity. A sounding rocket follows a suborbital trajectory and can produce several minutes of free fall. The period of free fall exists during its coast, after burn out, and before entering the atmosphere. Acceleration levels are usually at or below  $10^{-5} \text{ g}$ . NASA has employed many different sounding rockets for microgravity experiments. The most comprehensive series of launches, from 1975 to 1981, used SPAR (Space Processing Application Rocket) rockets for fluid physics, capillarity, liquid diffusion, containerless processing, and electrolysis experiments. The SPAR could lift 300 kg payloads into free fall parabolic trajectories lasting four to six minutes (Figures 5, 6).



## Mathematics Standards

Grades 5-8 ( $\Delta$ ); Grades 9-12 ( $\square$ )

- $\Delta \square$  Algebra
- $\Delta$  Computation and Estimation
  - $\square$  Conceptual Underpinnings of Calculus
  - $\square$  Discrete Mathematics
- $\Delta \square$  Mathematical Connections
- $\Delta \square$  Mathematics as Problem Solving
- $\Delta \square$  Mathematics as Reasoning
- $\Delta$  Number and Number Relationships

### Questions for Discussion

- How does the shuttle stay in orbit? Use the following two equations that describe the force acting on an object. The first equation represents the force of gravity acting on the shuttle.

$$F_1 = G \frac{m_e m_s}{r^2}$$

Where:

$F_1$  = Force of gravity acting on the shuttle

$G$  = Universal gravitational constant

$m_e$  = Mass of Earth

$m_s$  = Mass of the shuttle

$r$  = Distance from center of Earth to the shuttle

The second equation represents the force acting on the shuttle that causes a centripetal acceleration,  $v^2/r$ .

This is an expression of Newton's second law,  $F=ma$ .

$$F_2 = m_s \frac{v^2}{r}$$

$F_2$  = Force acting on the shuttle that causes uniform circular motion (with centripetal acceleration)

$v$  = Velocity of the shuttle

These two forces are equal:  $F_1 = F_2$

$$\frac{Gm_e m_s}{r^2} = \frac{m_s v^2}{r}$$

$$v^2 = \frac{Gm_e}{r}$$

$$v = \sqrt{\frac{Gm_e}{r}}$$

In order to stay in a circular orbit at a given distance from the center of Earth,  $r$ , the shuttle must travel at a precise velocity,  $v$ .

## Orbiting Spacecraft

Although airplanes, drop facilities, and small rockets can establish a microgravity environment, all of these laboratories share a common problem. After a few seconds or minutes of low-g, Earth gets in the way and the free fall stops. In spite of this limitation, we can learn much about fluid dynamics and mixing, liquid-gas surface interactions, and crystallization and macromolecular structure. To conduct longer term experiments (days, weeks, months, and years), we must travel into space and orbit Earth. Having more time available for experiments allows scientists to investigate slower processes and more subtle effects.

To establish microgravity conditions for long periods of time, one must first understand what keeps a spacecraft in orbit. Ask any group of students or adults **what keeps satellites and Space Shuttles in orbit** and you will probably get a variety of answers. Two common answers are: "The rocket engines keep firing to hold it up." and "There is no gravity in space."

Although the first answer is theoretically possible, the path followed by the spacecraft would technically not be an orbit. Other than the altitude involved and the specific means of exerting an upward force, little difference exists between a spacecraft with its engines constantly firing and an airplane flying around the world. A satellite could not carry enough fuel to maintain its altitude for more than a few minutes.

The second answer is also wrong. In a previous section, we discussed that Isaac Newton proved that the circular paths of the planets through space were due to gravity's presence, not its absence.

Newton expanded on his conclusions about gravity and hypothesized how an artificial satellite could be made to orbit Earth. He envisioned a very tall mountain extending above Earth's atmosphere so that friction with the air would not be a factor. He then imagined a cannon at the top of that mountain firing cannonballs parallel to the ground. Two forces acted upon each cannonball as it was fired. One force, the



explosion of the black powder, propelled the cannonball straight outward. If no other force were to act on the cannon ball, the shot would travel in a straight line and at a constant velocity. But Newton knew that a second force would act on the cannonball: the presence of gravity would cause the path of the cannonball to bend into an arc ending at Earth's surface (Figure 7).

Newton demonstrated how additional cannonballs would travel farther from the mountain each time if the cannon fired using more black powder. With each shot, the path would lengthen and soon the cannonballs would disappear over the horizon. Eventually, if one fired a cannon with enough energy, the cannonball would fall entirely around Earth and come back to its starting point. The cannonball would begin to orbit Earth. Provided no force other than gravity interfered with the cannonball's motion, it would continue circling Earth in that orbit.

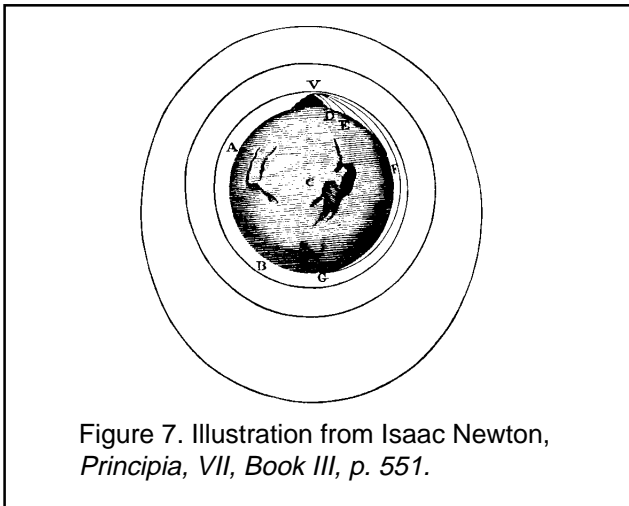


Figure 7. Illustration from Isaac Newton, *Principia*, VII, Book III, p. 551.

This is how the Space Shuttle stays in orbit. It launches into a trajectory that arcs above Earth so that the orbiter travels at the right speed to keep it falling while maintaining a constant altitude above the surface. For example, if the Shuttle climbs to a 320 kilometer high orbit, it must travel at a speed of about 27,740 kilometers per hour to achieve a stable orbit. **At that speed and altitude**, the Shuttle executes a falling path parallel to the curvature of Earth. Because the Space Shuttle travels in a state of free fall around Earth and due to the extremely low friction of the upper atmosphere, the Shuttle establishes a microgravity environment.

- How does the shuttle change its altitude? From a detailed equation relating the shuttle velocity with the shuttle altitude, one can obtain the following simple relationship for a circular orbit. Certain simplifying assumptions are made in developing this equation: 1) the radius of the shuttle orbit is nearly the same as the radius of Earth, and 2) the total energy of the shuttle in orbit is due to its kinetic energy,  $1/2 mv^2$ ; the change in potential energy associated with the launch is neglected.

$$\Delta r = \frac{\tau}{\pi} \Delta v$$

$\tau$  = orbital period, the time it takes the shuttle to complete one revolution around Earth

$$= \frac{2\pi r^{3/2}}{(Gm_e)^{1/2}}$$

$\Delta v$  = the change in shuttle velocity

$\Delta r$  = the change in shuttle altitude

For example:

Consider a shuttle in a circular orbit at 160 nm (296.3 km) altitude. Determine the new altitude caused by the shuttle firing a thruster that increases its velocity by 1m/s.

First, calculate the orbital period,  $\tau$ , from the above equation.

$$\begin{aligned} \tau &= \frac{2\pi(r_e + 2.96 \times 10^5 \text{ m})^{3/2}}{(Gm_e)^{1/2}} \\ &= \frac{2\pi(6.37 \times 10^6 \text{ m} + 2.96 \times 10^5 \text{ m})^{3/2}}{(6.67 \times 10^{-11} \frac{\text{m}^3}{\text{s}^2 \text{ kg}} \times 5.98 \times 10^{24} \text{ kg})^{1/2}} \\ &= 5.41 \times 10^3 \text{ s} \end{aligned}$$

Next, use the period and the applied velocity change to calculate the altitude change.

$$\begin{aligned} \Delta r &= \frac{\tau}{\pi} \Delta v \\ &= \frac{5.41 \times 10^3 \text{ s}}{\pi} (1 \text{ m/s}) \\ &= 1.72 \times 10^3 \text{ m} \end{aligned}$$

In order to make the orbit circular at the new altitude, the shuttle needs to apply the same  $\Delta v$  at the other side of the orbit.

In the discussion and example just given, we state that the equations given are simple approximations of more complex relationships between shuttle velocity and altitude. The more complex equations are used by the shuttle guidance and navigation teams who track the shuttles' flights. But the equations given here can be used for quick approximations of the types of thruster firings needed to achieve certain altitude changes. This is helpful when an experiment team may want to request an altitude change. Engineers supporting the experiment teams can determine approximately how much propellant would be required for such an altitude change and whether enough would be left for the required de-orbit burns. In this way, the engineers and experiment teams can see if their request is realistic and if it has any possibility of being implemented.

### Mathematics Standards

Grades 5-8 ( $\Delta$ ); Grades 9-12 ( $\square$ )

- $\square$  Functions
- $\Delta \square$  Mathematical Connections
- $\Delta \square$  Mathematics as Communication
- $\Delta$  Patterns and Functions

When microgravity researchers study the effects of atmospheric friction, shuttle motion, and other related acceleration phenomena on their experiments, three concepts play an important role: Right-hand rule, Coordinate systems, and Frames of reference.

Orbiting spacecraft provide ideal laboratories for microgravity research. As on airplanes, scientists can fly with the experiments that are on the spacecraft. Because scientists are there to tend the experiments, the experiments do not have to be fully automatic in operation. A malfunction in an experiment conducted with a drop tower or small rocket means a loss of data or complete failure. In orbiting spacecraft, crew members can make repairs so that there is little or no loss of data. They can also make on-orbit modifications in experiments to gather more diverse data. Perhaps the greatest advantage of orbiting spacecraft for microgravity research is the amount of time during which microgravity conditions can be achieved. Experiments lasting for more than two weeks are possible with the Space Shuttle. When the International Space Station becomes operational, the time available for experiments will stretch to months. The International Space Station will provide a manned microgravity laboratory facility unrivaled by any on Earth.

#### "Microgravity Room"

One of the common questions asked by visitors to the NASA Johnson Space Center in Houston, Texas is, "Where is the room where a button is pushed and gravity goes away so that astronauts float?" No such room exists because gravity can never be made to go away. The misconception comes from the television pictures that NASA takes of astronauts training in the KC-135 and from underwater training pictures. Astronauts scheduled to wear spacesuits for extravehicular activities train in the Weightless Environment Training Facility (WET F). The WET F is a swimming pool large enough to hold a Space Shuttle payload bay mock-up and mock-ups of satellites and experiments. Astronauts add heavy weights to their spacesuits, which are filled with air, to achieve neutral buoyancy in the water. The facility provides an excellent simulation of what it is like to work in space, with two exceptions: in the pool astronauts can swim with hand and leg motions, and if they drop a hand tool, it falls to the bottom.





# NASA Resources for Educators

**NASA's Central Operation of Resources for Educators (CORE)** was established for the national and international distribution of NASA-produced educational materials in audiovisual format. Educators can obtain a catalog and an order form by one of the following methods:

- NASA CORE  
Lorain County Joint Vocational School  
15181 Route 58 South  
Oberlin, OH 44074
- Phone (216) 774-1051, Ext. 249 or 293
- Fax (216) 774-2144
- E-mail: [nasaco@leeca8.leeca.ohio.gov](mailto:nasaco@leeca8.leeca.ohio.gov)
- Home Page: <http://spacelink.msfc.nasa.gov/CORE>

## Educator Resource Center Network

To make additional information available to the education community, the NASA Education Division has created the NASA Educator Resource Center (ERC) network. ERCs contain a wealth of information for educators: publications, reference books, slide sets, audio cassettes, videotapes, telelecture programs, computer programs, lesson plans, and teacher guides with activities. Educators may preview, copy, or receive NASA materials at these sites. Because each NASA Field Center has its own areas of expertise, no two ERCs are exactly alike. Phone calls are welcome if you are unable to visit the ERC that serves your geographic area. A list of the centers and the regions they serve includes:

AK, AZ, CA, HI, ID, MT, NV, OR,  
UT, WA, WY  
NASA Educator Resource Center  
Mail Stop 253-2  
**NASA Ames Research Center**  
Moffett Field, CA 94035-1000  
Phone: (415) 604-3574

CT, DE, DC, ME, MD, MA, NH,  
NJ, NY, PA, RI, VT  
NASA Educator Resource Laboratory  
Mail Code 130.3  
**NASA Goddard Space Flight Center**  
Greenbelt, MD 20771-0001  
Phone: (301) 286-8570

CO, KS, NE, NM, ND, OK, SD, TX  
NASA Educator Resource Room  
Mail Code AP-2  
**NASA Johnson Space Center**  
2101 NASA Road One  
Houston, TX 77058-3696  
Phone: (281) 483-8696

FL, GA, PR, VI  
NASA Educator Resource Laboratory  
Mail Code ERL  
**NASA Kennedy Space Center**  
Kennedy Space Center, FL 32899-0001  
Phone: (407) 867-4090

KY, NC, SC, VA, WV  
Virginia Air and Space Museum  
NASA Educator Resource Center for  
**NASA Langley Research Center**  
600 Settler's Landing Road  
Hampton, VA 23669-4033  
Phone: (757) 727-0900 x 757

IL, IN, MI, MN, OH, WI  
NASA Educator Resource Center  
Mail Stop 8-1  
**NASA Lewis Research Center**  
21000 Brookpark Road  
Cleveland, OH 44135-3191  
Phone: (216) 433-2017

**Regional Educator Resource Centers (RERCs)** offer more educators access to NASA educational materials. NASA has formed partnerships with universities, museums, and other educational institutions to serve as RERCs in many states. A complete list of RERCs is available through CORE, or electronically via NASA Spacelink.

**NASA On-line Resources for Educators** provide current educational information and instructional resource materials to teachers, faculty, and students. A wide range of information is available, including science, mathematics, engineering, and technology education lesson plans, historical information related to the aeronautics and space program, current status reports on NASA projects, news releases, information on NASA educational programs, useful software and graphics files. Educators and students can also use NASA resources as learning tools to explore the Internet, to access information about educational grants, to interact with other schools that are already on-line, to participate in on-line interactive projects, and to communicate with NASA scientists, engineers, and other team members to experience the excitement of real NASA projects.

Access these resources through the NASA Education Home Page: <http://www.hq.nasa.gov/education>

or, for more information, send an e-mail to: [comments@spacelink.msfc.nasa.gov](mailto:comments@spacelink.msfc.nasa.gov)

**NASA Television (NTV)** is the Agency's distribution system for live and taped programs. It offers the public a front-row seat for launches and missions, as well as informational and educational programming, historical documentaries, and updates on the latest developments in aeronautics and space science. NTV is transmitted on GE-2, Transponder 9C at 85 degrees West longitude, vertical polarization, with a frequency of 3,880 megahertz, audio on 6.8 megahertz, or through collaborating distance learning networks and local cable providers.

Apart from live mission coverage, regular NASA Television programming includes a News Video File from noon to 1:00 pm, a NASA History File from 1:00 to 2:00 pm, and an Education File from 2:00 to 3:00 pm (all times Eastern). This sequence is repeated at 3:00 pm, 6:00 pm, and 9:00 pm, Monday through Friday. The NTV Education File features programming for teachers and students on science, mathematics, and technology, including the *NASA... On the Cutting Edge* Education Satellite Videoconference Series. The videoconferences include NASA scientists, astronauts, and education specialists presenting aeronautics and Earth and space science topics of interest to teachers and students of grades 5-12. The series is free to registered educational institutions. The videoconferences and all NASA Television programming may be videotaped for later use.

For more information on NASA Television, contact:  
NASA Headquarters, Code P-2, NASA TV, Washington, DC 20546-0001 Phone: (202) 358-3572  
Home Page: <http://www.hq.nasa.gov/office/pao/ntv.html>

For more information about the Education Satellite Videoconference Series, contact: Videoconference Producer, NASA Teaching From Space Program, 308 CITD, Room A, Oklahoma State University, Stillwater, OK 74078-8089  
E-mail: [edge@aesp.nasa.okstate.edu](mailto:edge@aesp.nasa.okstate.edu)  
Home Page: <http://www.okstate.edu/aesp/VC.html>

**How to Access NASA's Education Materials and Services, EP-1996-11-345-HQ** This brochure serves as a guide to accessing a variety of NASA materials and services for educators. Copies are available through the ERC network, or electronically via NASA Spacelink.



# TEACHER REPLY CARD

## The Mathematics of Microgravity Educational Brief

To achieve America's goals in Educational Excellence, it is NASA's mission to develop supplementary instructional materials and curricula in science, mathematics, and technology. NASA seeks to involve the educational community in the development and improvement of these materials. Your evaluation and suggestions are vital to continually improving NASA educational materials.

**Please take a moment** to respond to the statements and questions below. You can submit your response through the Internet or by mail. Send your reply to the following Internet address:

[http://ednet.gsfc.nasa.gov/edcats/educational\\_brief](http://ednet.gsfc.nasa.gov/edcats/educational_brief)

You will then be asked to enter your data at the appropriate prompt.

Otherwise, please tear off the reply card, place in an envelope, and return by mail to: NASA Headquarters, Education Division, Mail Code FE, Washington, DC 20546-0001. Thank you.

1. With what grades did you use the educational brief?

Number of **Teachers/Faculty**:

\_\_\_\_ K-4    \_\_\_\_ 5-8    \_\_\_\_ 9-12    \_\_\_\_ Community College    College/University  
\_\_\_\_ Graduate  
\_\_\_\_ Undergraduate

Number of **Students**:

\_\_\_\_ K-4    \_\_\_\_ 5-8    \_\_\_\_ 9-12    \_\_\_\_ Community College    College/University  
\_\_\_\_ Graduate  
\_\_\_\_ Undergraduate

Number of **Others**:

\_\_\_\_ Administrators/Staff    \_\_\_\_ Parents    \_\_\_\_ Professional Groups    \_\_\_\_ Civic Groups  
\_\_\_\_ General Public    Other \_\_\_\_\_  
\_\_\_\_\_

2. What is your 9-digit zip code? \_\_\_\_\_

3. How was the quality of this educational brief?

Excellent     Good     Average     Poor     Very Poor

4. How did you use this educational brief?

- |  |   |
|--|---|
| <input type="checkbox"/> Background Information              | <input type="checkbox"/> Critical Thinking Tasks    |
| <input type="checkbox"/> Demonstrate NASA Materials          | <input type="checkbox"/> Demonstration              |
| <input type="checkbox"/> Group Discussions                   | <input type="checkbox"/> Hands-On Activities        |
| <input type="checkbox"/> Integration Into Existing Curricula | <input type="checkbox"/> Interdisciplinary Activity |
| <input type="checkbox"/> Lecture                             | <input type="checkbox"/> Science and Mathematics    |
| <input type="checkbox"/> Team Activities                     | Standards Integration                               |
| <input type="checkbox"/> Other: Please specify:              |   |

\_\_\_\_\_

5. What features of this educational brief did you find particularly helpful?

\_\_\_\_\_  
\_\_\_\_\_  
\_\_\_\_\_  
\_\_\_\_\_

6. How can we make this educational brief more effective for you?

\_\_\_\_\_  
\_\_\_\_\_  
\_\_\_\_\_  
\_\_\_\_\_

7. Additional comments:

\_\_\_\_\_  
\_\_\_\_\_  
\_\_\_\_\_

Today's Date: \_\_\_\_\_