BUTTERWORTH FILTER DESIGN

Objective

The main purpose of this laboratory exercise is to learn the procedures for designing Butterworth filters. In addition, the theory behind these procedures will be discussed in recitation and the hands-on methods practiced in the laboratory. This exercise will focus on the fourth-order filter but the methods are valid regardless of the order of the filter. In this lab only even order filters will be considered.

Background

Read pgg.728-738 Hambley and the handout. The Butterworth and Chebyshev filters are high order filter designs which have significantly different characteristics, but both can be realized by using simple first order or biquad stages cascaded together to achieve the desired order, passband response, and cut-off frequency. The Butterworth filter has a maximally flat response, i.e., no passband ripple and a roll-off of -20dB per pole. In the Butterworth scheme the designer is usually optimizing the flatness of the passband response at the expense of roll-off. The Chebyshev filter displays a much steeper roll-off, but the gain in the passband is not constant. The Chebyshev filter is characterized by a significant passband ripple that often can be ignored. The designer in this case is optimizing roll-off at the expense of passband ripple.

Both filter types can be implemented using the simple Sallen-Key configuration. The Sallen-Key design (shown in Figure A-1) is a biquadratic or biquad type filter, meaning there are two poles defined by the circuit transfer function

$$H(s) = \frac{H_0}{(s/\omega_0)^2 + (1/Q)(s/\omega_0) + 1} \quad , \tag{1}$$

where H_0 is the DC gain of the biquad circuit and is a function only of the ratio of the two resistors connected to the negative input of the opamp. The quantity Q is called the quality factor and is a direct measure of the flatness of the passband (in particular, a large value of Q indicates peaking at the edge of the passband.) When $C_1 = C_2 = C$ and $R_1 = R_2 = R$ for the Sallen-Key design, the value of Q depends exclusively on the gain of the op-amp stage and the cut-off frequency depends on R and C, or

$$Q = \frac{1}{(3 - H_0)} \quad , \qquad \text{and} \tag{2}$$

$$f_C = \frac{1}{2\pi RC} \quad . \tag{3}$$

Notice that the quality factor Q and the DC gain H_0 cannot be independently set in the Sallen-Key circuit.

Choice of the value of Q depends on the desired characteristics of the filter. In particular the designer must consider both the quality and the stability of the circuit. Calculations with different values of Q demonstrate that for the second order case the flattest response occurs when $Q = 0.707$, hence the maximally flat response of the Butterworth filter will also be realized when $Q = 0.707$. In addition the circuit is only stable when $H_0 < 3$. Higher values will result in poles in the right half-plane and an unstable circuit.

The Butterworth filter is an optimal filter with maximally flat response in the passband. The magnitude of the frequency response of this family of filters can be written as

$$|H(f)| = \frac{H_0}{\sqrt{1 + (f / f_C)^{2n}}} \quad , \tag{4}$$

where n is the order of the filter and f_c is the cutoff frequency . For a single biquad section, i.e., $n = 2$, the gain for which Q will equal 0.707, and hence yield a Butterworth type response, is found to be 1.586 from equation (2).

In the case of a fourth order Butterworth filter (n = 4), the correct response can be achieved by cascading two biquad stages. Each stage must be characterized by a specific value of gain (or Q) that achieves both a flat response and stability. For a fourth order Butterworth filter, the Q factors of the two stages must be $Q = 0.541$ and $Q = 1.307$. From (2) it follows that one stage must have a gain of 1.152 and the other stage a gain of 2.235. As a result the overall DC gain of the fourth-order Butterworth realized with two biquad stages will be equal to the product of the DC gain of the two stages, i.e., 2.575. The gain values required to cascade biquad sections to achieve even-order filters are given in Table A-I. Note that the number of biquad stages needed to realize a filter of order n is $n/2$.

Other types of filters such as high pass, and bandpass filters can be designed the same way, i.e., by cascading the appropriate filter sections to achieve the desired bandpass and roll-off. The high-pass dual to the Sallen-Key low pass filter can be realized by simply exchanging the positions of R and C in the circuit of Figure A-1.

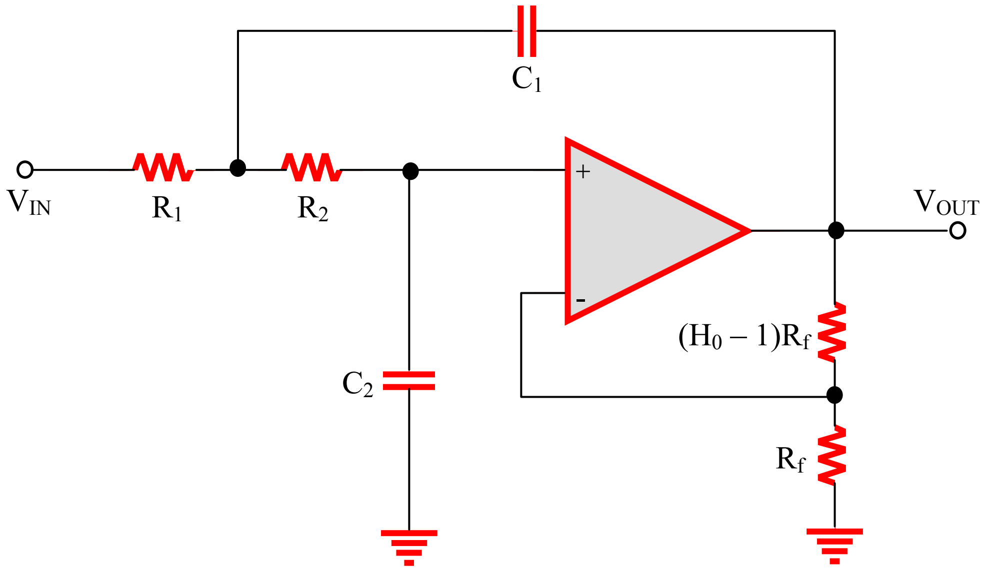
Prelab

1. Design the lowpass, bandpass and highpass filters according to the specifications on the class web page.
2. What is the passband gain of the filters?
3. Simulate the filters using Spice. Produce Bode plots for magnitude and phase for the three filters. Apply to the lowpass filter a 1V square wave and simulate the response of the lowpass filter. Do this for three cases: square wave frequency equal to 0.1 f_c, 0.5 f_c, and f_c, where f_c is the cutoff frequency of the lowpass filter.

Lab Project

1. Build the Butterworth filters you designed for the Prelab and measure their frequency response.
2. Apply a 1 volt sinusoidal input and observe both the input and output waveforms for ten frequencies equally spaced on the logarithmic scale and measure the response magnitude and phase of each.
3. Compare the response to the model predictions from the prelab.

Appendix



For $R_1 = R_2$ and $C_1 = C_2$

$$H(s) = \frac{H_0}{(RC)^2 s^2 + RCs(3 - H_0) + 1} \qquad Q = \frac{1}{(3 - H_0)}$$

$$\omega_0 = \frac{1}{RC} \quad or \quad f_C = \frac{1}{2\pi RC}$$

Figure A-1 The Sallen-Key low pass filter

Poles	Butterworth (Gain)	Chebyshev (0.5dB) λ_n	Chebyshev (0.5dB) Gain	Chebyshev (2.0dB) λ_n	Chebyshev (2.0dB) Gain
2	1.586	1.231	1.842	0.907	2.114
4	1.152	0.597	1.582	0.471	1.924
	2.235	1.031	2.660	0.964	2.782
6	1.068	0.396	1.537	0.316	1.891
	1.586	0.768	2.448	0.730	2.648
	2.483	1.011	2.846	0.983	2.904
8	1.038	0.297	1.522	0.238	1.879
	1.337	0.599	2.379	0.572	2.605
	1.889	0.861	2.711	0.842	2.821
	2.610	1.006	2.913	0.990	2.946