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29
Active Filters

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29.1 Synthesis of Low-Pass Forms

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Passive and Active Filters

There are formal definitions of activity and passivity in electronics, but it is sufficient to observe that passive filters are built from passive components; resistors, capacitors, and inductors are the commonly encountered building blocks although distributed RC components, quartz crystals, and surface acoustic wave devices are used in filters working in the high-megahertz regions. **Active filters** also use resistors and capacitors, but the inductors are replaced by active devices capable of producing power gain. These devices can range from single transistors to integrated circuit (IC) -controlled sources such as the operational amplifier (op amp), and more exotic devices, such as the operational transconductance amplifier (OTA), the generalized impedance converter (GIC), and the frequency-dependent negative resistor (FDNR).

The theory of filter synthesis, whether active or passive, involves the determination of a suitable circuit topology and the computation of the circuit component values within the topology, such that a required network response is obtained. This response is most commonly a voltage transfer function (VTF) specified in the frequency domain. Circuit analysis will allow the performance of a filter to be evaluated, and this can be done by obtaining the VTF, $H(s)$, which is, in general, a rational function of s, the complex frequency variable. The *poles* of a VTF correspond to the roots of its denominator polynomial. It was established early in the history of filter theory that a network capable of yielding complex-conjugate transfer function (TF) pole-pairs is required to achieve high selectivity. A highly selective network is one that gives a rapid transition between passband and stopband regions of the frequency response. Figure 29.1(a) gives an example of a passive low-pass LCR ladder network capable of producing a VTF with the necessary pole pattern.

The network of Fig. 29.1(a) yields a VTF of the form

$$H(s) = \frac{V_{\text{out}}(s)}{V_{\text{in}}(s)} = \frac{1}{a_5 s^5 + a_4 s^4 + a_3 s^3 + a_2 s^2 + a_1 s + a_0} \tag{29.1}$$

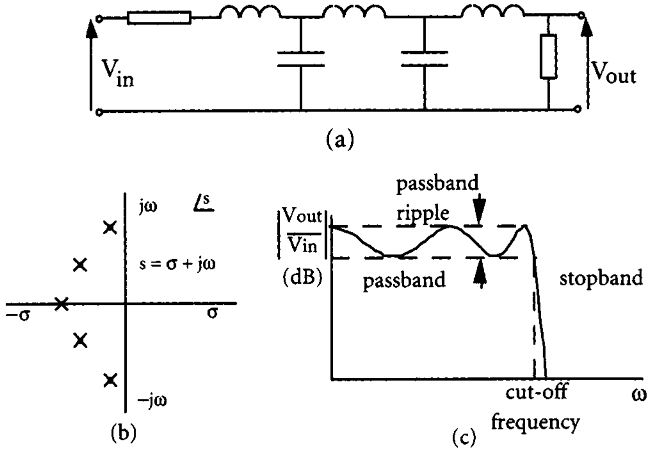

FIGURE 29.1 (a) Passive LCR filter; (b) typical pole plot; (c) typical frequency response.

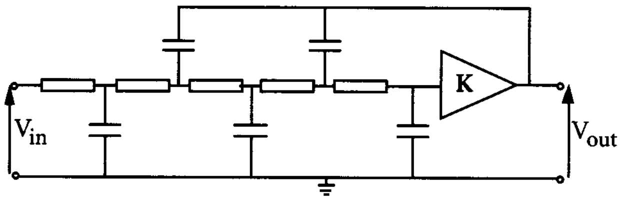

FIGURE 29.2 RC-active filter equivalent to circuit of Fig. 29.1(a).

Figure 29.1(b) shows a typical pole plot for the fifth-order VTF produced by this circuit. Figure 29.1(c) gives a sample sinusoidal steady-state frequency response plot. The frequency response is found by setting $s = j\omega$ in Eq. (29.1) and taking $|H(j\omega)|$. The LCR low-pass ladder structure of Fig. 29.1(a) can be altered to higher or lower order simply by adding or subtracting reactances, preserving the series-inductor/shunt-capacitor pattern. In general terms, the higher the filter order, the greater the selectivity.

This simple circuit structure is associated with a well-established design theory and might appear the perfect solution to the filter synthesis problem. Unfortunately, the problems introduced by the use of the inductor as a circuit component proved a serious difficulty from the outset. Inductors are intrinsically nonideal components, and the lower the frequency range of operation, the greater these problems become. Problems include significant series resistance associated with the physical structure of the inductor as a coil of wire, its ability to couple by electromagnetic induction into fields emanating from external components and sources and from other inductors within the filter, its physical size, and potential mechanical instability. Added to these problems is the fact that the inductor tends not to be an off-the-shelf component but has instead to be fabricated to the required value as a custom device. These serious practical difficulties created an early pressure to develop alternative approaches to electrical filtering. After the emergence of the electronic amplifier based on vacuum rules, it was discovered that networks involving resistors, capacitors, and amplifiers—*RC-active filters*—were capable of producing TFs exactly equivalent to those of LCR ladders. Figure 29.2 shows a single-amplifier multiloop ladder structure that can produce a fifth-order response identical to that of the circuit of Fig. 29.1(a).

The early active filters, based as they were on tube amplifiers, did not constitute any significant advance over their passive counterparts. It required the advent of solid-state active devices to make the RC-active filter a viable alternative. Over the subsequent three decades, active filter theory has developed to an advanced state, and this development continues as new IC technologies create opportunities for novel network structures and applications.

Active Filter Classification and Sensitivity

There are two major approaches to the synthesis of RC-active filters. In the first approach, a TF specification is factored into a product of second-order terms. Each of these terms is realized by a separate RC-active

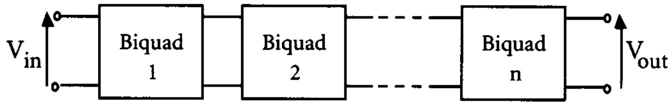

FIGURE 29.3 Biquad cascade realizing high-order filter.

subnetwork designed to allow for non-interactive interconnection. The subnetworks are then connected in cascade to realize the required overall TF, as shown in Fig. 29.3. A first-order section is also required to realize odd-order TF specifications. These second-order sections may, depending on the exact form of the overall TF specification, be required to realize numerator terms of up to second order. An RC-active network capable of realizing a biquadratic TF (that is, one whose numerator *and* denominator polynomials are second-order) is called a **biquad**.

This scheme has the advantage of design ease since simple equations can be derived relating the components of each section to the coefficients of each factor in the VTF. Also, each biquad can be independently adjusted relatively easily to give the correct performance. Because of these important practical merits, a large number of alternative biquad structures have been proposed, and the newcomer may easily find the choice overwhelming.

The second approach to active filter synthesis involves the use of RC-active circuits to simulate passive LCR ladders. This has two important advantages. First, the design process can be very straightforward: the wealth of design data published for passive ladder filters (see Further Information) can be used directly so that the sometimes difficult process of component value synthesis from specification is eliminated. Second, the LCR ladder offers optimal **sensitivity** properties [Orchard, 1966], and RC-active filters designed by ladder simulation share the same low sensitivity features. Chapter 4 of Bowron and Stephenson [1979] gives an excellent introduction to the formal treatment of circuit sensitivity.

Sensitivity plays a vital role in the characterization of RC-active filters. It provides a measure of the extent to which a change in the value of any given component affects the response of the filter. High sensitivity in an RC-active filter should also alert the designer to the possibility of oscillation. A nominally stable design will be unstable in practical realization if sensitivities are such that component value errors cause one or more pairs of poles to migrate into the right half plane. Because any practical filter will be built with components that are not exactly nominal in value, sensitivity information provides a practical and useful indication of how different filter structures will react and provides a basis for comparison.

Cascaded Second-Order Sections

This section will introduce the cascade approach to active filter design. As noted earlier, there are a great many second-order RC-active sections to choose from, and the present treatment aims only to convey some of the main ideas involved in this strategy. The references provided at the end of this section point the reader to several comprehensive treatments of the subject.

Sallen and Key Section

This is an early and simple example of a second-order section building block [Sallen and Key, 1955]. It remains a commonly used filter despite its age, and it will serve to illustrate some key stages in the design of all such RC-active sections. The circuit is shown in Fig. 29.4. A straightforward analysis of this circuit yields a VTF

$$H(s) = \frac{K\dfrac{1}{C_1C_2R_1R_2}}{s^2 + s\left[\dfrac{1}{C_2R_2} + \dfrac{1}{C_2R_1} + \dfrac{1-K}{C_1R_1}\right] + \dfrac{1}{C_1C_2R_1R_2}} \tag{29.2}$$

This is an all-pole low-pass form since the numerator involves only a constant term.

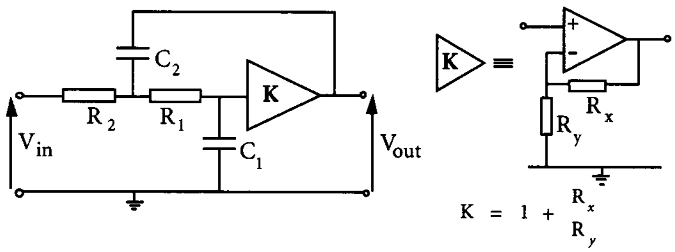

FIGURE 29.4 Sallen and Key second-order filter section.

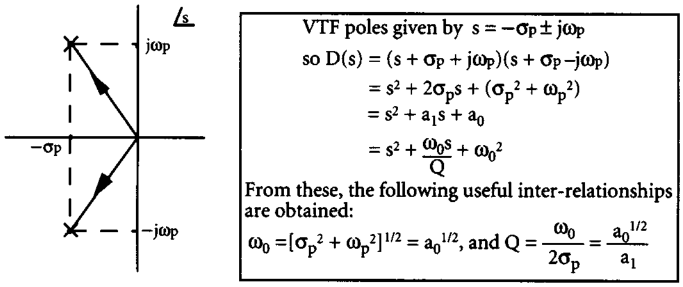

FIGURE 29.5 VTF pole relationships.

Specifications for an all-pole second-order section may arise in coefficient form, where the required *s*-domain VTF is given as

$$H(s) = \frac{k}{s^2 + a_1 s + a_0} \tag{29.3}$$

or in Q-ω_0 standard second-order form

$$H(s) = \frac{k}{s^2 + \dfrac{\omega_0}{Q} s + \omega_0} \tag{29.4}$$

Figure 29.5 shows the relationship between these VTF forms.

As a design example, the VTF for an all-pole fifth-order Chebyshev filter with 0.5-dB passband ripple [see Fig. 29.1(c)] has the factored-form denominator

$$D(s) = (s + 0.36232)(s^2 + 0.22393\,s + 1.0358)(s^2 + 0.58625s + 0.47677) \tag{29.5}$$

Taking the first of the quadratic factors in Eq. (29.5) and comparing like coefficients from Eq. (29.2) gives the following design equations:

$$\frac{1}{C_1C_2R_1R_2} = 1.0358; \qquad \frac{1}{C_2R_2} + \frac{1}{C_2R_1} + \frac{1-K}{C_1R_1} = 0.22393 \tag{29.6}$$

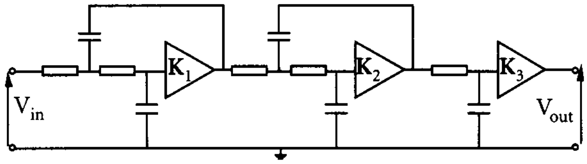

FIGURE 29.6 Form of fifth-order Sallen and Key cascade.

Clearly, the designer has some degrees of freedom here since there are two equations in five unknowns. Choosing to set both (normalized) capacitor values to unity, and fixing the dc stage gain $K = 5$, gives

$$C_1 = C_2 = 1\text{F};\ R_1 = 1.8134\ \Omega;\ R_2 = 1.3705\ \Omega;\ R_x = 4\ \Omega;\ R_y = 1\ \Omega$$

Note that Eq. (29.5) is a normalized specification giving a filter cut-off frequency of 1 rad s^{-1}. These normalized component values can now be denormalized to give a required cut-off frequency and practical component values. Suppose that the filter is, in fact, required to give a cut-off frequency $f_c = 1$ kHz. The necessary shift is produced by multiplying all the capacitors (leaving the resistors fixed) by the factor ω_N/ω_D where ω_N is the normalized cut-off frequency (1 rad s^{-1} here) and ω_D is the required denormalized cut-off frequency ($2\pi \times 1000$ rad s^{-1}). Applying this results in denormalized capacitor values of 159.2 µF. A useful rule of thumb [Waters, 1991] advises that capacitor values should be on the order of magnitude of $(10/f_c)$ µF, which suggests that the capacitors should be further scaled to around 10 nF. This can be achieved without altering of the filter's f_c, by means of the impedance scaling property of electrical circuits. Providing all circuit impedances are scaled by the same amount, current and voltage TFs are preserved. In an RC-active circuit, this requires that all resistances are multiplied by some factor while all capacitances are divided by it (since capacitive impedance is proportional to $1/C$). Applying this process yields final values as follows:

$$C_1,\ C_2 = 10\ \text{nF};\ R_1 = 29.86\ \text{k}\Omega;\ R_2 = 21.81\ \text{k}\Omega;\ R_x = 63.66\ \text{k}\Omega;\ R_y = 15.92\ \text{k}\Omega$$

Note also that the dc gain of each stage, $|H(0)|$, is given by K [see Eq. (29.2) and Fig. 29.4] and, when several stages are cascaded, the overall dc gain of the filter will be the product of these individual stage gains. This feature of the Sallen and Key structure gives the designer the ability to combine easy-to-manage amplification with prescribed filtering.

Realization of the complete fifth-order Chebyshev VTF requires the design of another second-order section to deal with the second quadratic term in Eq. (29.5), together with a simple circuit to realize the first-order term arising because this is an odd-order VTF. Figure 29.6 shows the form of the overall cascade. Note that the op amps at the output of each stage provide the necessary interstage isolation. It is finally worth noting that an extended single-amplifier form of the Sallen and Key network exists—the circuit shown in Fig. 29.2 is an example of this—but that the saving in op amps is paid for by higher component spreads, sensitivities, and design complexity.

State-Variable Biquad

The simple Sallen and Key filter provides only an all-pole TF; many commonly encountered filter specifications are of this form—the Butterworth and Chebyshev approximations are notable examples—so this is not a serious limitation. In general, however, it will be necessary to produce sections capable of realizing a second-order denominator together with a numerator polynomial of up to second-order:

$$H(s) = \frac{b_2 s^2 + b_1 s + b_0}{s^2 + a_1 s + a_0} \qquad (29.7)$$

The other major filter approximation in common use—the elliptic (or Cauer) function filter—involves quadratic numerator terms in which the b_1 coefficient in Eq. (29.7) is missing. The resulting numerator polynomial, of the form $b_2 s^2 + b_0$, gives rise to s-plane zeros on the $j\omega$ axis corresponding to points in the stopband of the sinusoidal frequency response where the filter's transmission goes to zero. These notches or *transmission zeros* account for the elliptic's very rapid transition from passband to stopband and, hence, its optimal selectivity.

A filter structure capable of producing a VTF of the form of Eq. (29.7) was introduced as a state-variable realization by its originators [Kerwin et al., 1967]. The structure comprises two op amp integrators and an op amp summer connected in a loop and was based on the integrator-summer analog computer used in control/analog systems analysis, where system state is characterized by some set of so-called state variables. It is also often referred to as a ring-of-three structure. Many subsequent refinements of this design have appeared (Schaumann et al., [1990] gives a useful treatment of some of these developments) and the state-variable biquad has achieved considerable popularity as the basis of many commercial universal packaged active filter building blocks. By selecting appropriate chip/package output terminals, and with the use of external trimming components, a very wide range of filter responses can be obtained.

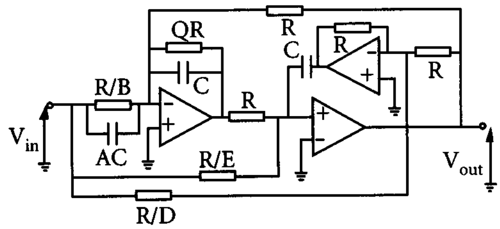

FIGURE 29.7 Circuit schematic for state-variable biquad.

Figure 29.7 shows a circuit developed from this basic state-variable network and described in Schaumann et al. [1990]. The circuit yields a VTF

$$H(s) = \frac{V_{out}(s)}{V_{in}(s)} = -\frac{As^2 + \omega_0(B - D)s + E\omega_0^2}{s^2 + \frac{\omega_0}{Q}s + \omega_0^2}, \text{ with } \omega_0 \triangleq 1/RC \tag{29.8}$$

By an appropriate choice of the circuit component values, a desired VTF of the form of Eq. (29.8) can be realized.

Consider, for example, a specification requirement for a second-order elliptic filter cutting off at 10 kHz. Assume that a suitable normalized (1 rad/s) specification for the VTF is

$$H(s) = -\frac{0.15677(s^2 + 7.464)}{s^2 + 0.9989s + 1.1701} \tag{29.9}$$

From Eq. (29.8) and Eq. (29.9), and referring to Fig. 29.7, normalized values for the components are computed as follows. As the s term in the numerator is to be zero, set $B = D = 0$ (which obtains if resistors R/B and R/D are simply removed from the circuit). Setting $C = 1$ F gives the following results:

$$\text{AC} = 0.15677\text{F}; \text{R} = 1/\text{C}\omega_0 = 0.92446\ \Omega; \text{QR} = 1.08290\ \Omega; \text{R/E} = 0.92446\ \Omega$$

Removing the normalization and setting $C = (10/10\text{ k})\ \mu\text{F} = 1$ nF requires capacitors to be multiplied by 10^{-9} and resistors to be multiplied by 15.9155×10^3. Final denormalized component values for the 10-kHz filter are thus:

$$\text{C} = 1\ \text{nF}; \text{AC} = 0.15677\ \text{nF}; \text{R} = \text{R/E} = 14.713\ \text{k}\Omega; \text{QR} = 17.235\ \text{k}\Omega$$

Passive Ladder Simulation

As for the biquad approach, numerous different ladder-based design methods have been proposed. Two representative schemes will be considered here: inductance simulation and ladder transformation.

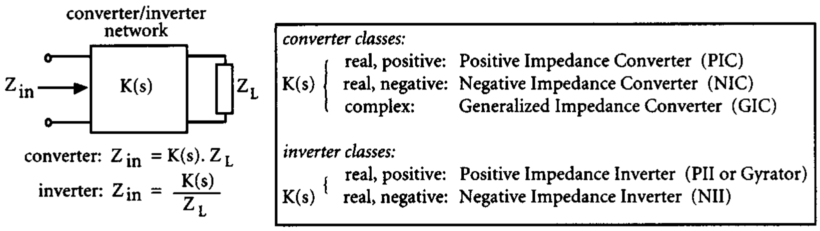

FIGURE 29.8 Generic impedance converter/inverter networks.

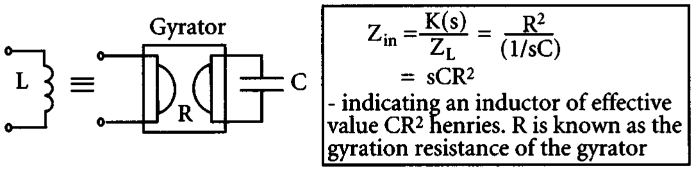

FIGURE 29.9 Gyrator simulation of an inductor.

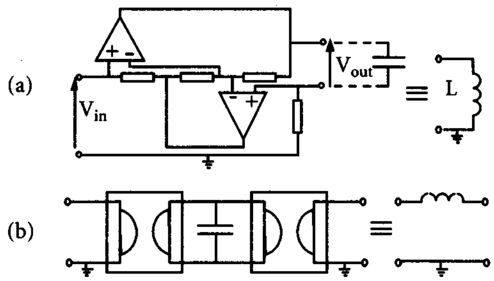

FIGURE 29.10 (a) Practical gyrator and (b) simulation of floating inductor. (Source: A. Antoniou, Proc. IEE, vol. 116, pp. 1838–1850, 1969. With permission.)

Inductance Simulation

In the inductance simulation approach, use is made of impedance converter/inverter networks. Figure 29.8 gives a classification of the various generic forms of device. The NIC enjoyed prominence in the early days of active filters but was found to be prone to instability. Two classes of device that have proved more useful in the longer term are the GIC and the gyrator.

Figure 29.9 introduces the symbolic representation of a gyrator and shows its use in simulating an inductor.

The gyrator can conveniently be realized by the circuit of Fig. 29.10(a), but note that the simulated inductor is grounded at one end. This presents no problem in the case of high-pass filters and other forms requiring a grounded shunt inductor but is not suitable for the low-pass filter. Figure 29.10(b) shows how a pair of back-to-back gyrators can be configured to produce a floating inductance, but this involves four op amps per inductor.

The next section will introduce an alternative approach that avoids the op amp count difficulty associated with simulating the floating inductors directly.

Ladder Transformation

The other main approach to the RC-active simulation of passive ladders involves the transformation of a prototype ladder into a form suitable for active realization. A most effective method of this class is based on the use of the Bruton transformation [Bruton, 1969], which involves the complex impedance scaling of a prototype passive LCR ladder network. All prototype circuit impedances $Z(s)$ are transformed to $Z_T(s)$ with

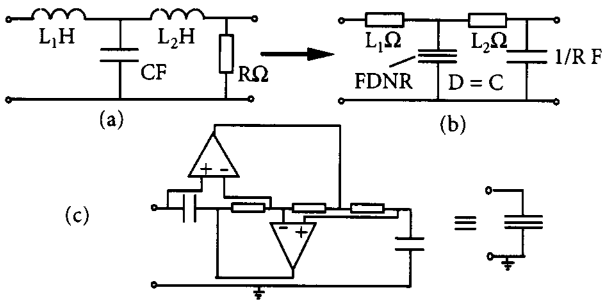

FIGURE 29.11 FDNR active filter.

$$Z_T(s) = \frac{K}{s} \cdot Z(s) \tag{29.10}$$

where K is a constant chosen by the designer and which provides the capacity to scale component values in the final filter. Since impedance transformations do not affect voltage and current transfer ratios, the VTF remains unaltered by this change. The Bruton transformation is applied directly to the elements in the prototype network, and it follows from Eq. (29.10) that a resistance R transforms into a capacitance $C = K/R$, while an inductance L transforms into a resistance $R = KL$. The elimination of inductors in favor of resistors is the key purpose of the Bruton transform method. Applying the Bruton transform to a prototype circuit capacitance C gives

$$Z_T(s) = \frac{K}{s} \cdot \frac{1}{sC} = \frac{K}{s^2 C} = \frac{1}{s^2 D} \tag{29.11}$$

where $D = C/K$ is the parameter value of a new component produced by the transformation, which is usually referred to as a frequency-dependent negative resistance (FDNR). This name results from the fact that the sinusoidal steady-state impedance $Z_T(j\omega) = -(1/\omega^2 D)$ is frequency-dependent, negative, and real, hence, resistive. In practice, the FDNR elements are realized by RC-active subnetworks using op amps, normally two per FDNR. Figure 29.11(a) and (b) shows the sequence of circuit changes involved in transforming from a third-order LCR prototype ladder to an FDNR circuit. Figure 29.11(c) gives an RC-active realization for the FDNR based on the use of a GIC, introduced in the previous subsection.

Active Filters for ICs

It was noted earlier that the advent of the IC op amp made the RC-active filter a practical reality. A typical state-of-the-art 1960–70s active filter would involve a printed circuit board-mounted circuit comprising discrete passive components together with IC op amps. Also appearing at this time were hybrid implementations, which involve special-purpose discrete components and op amp ICs interconnected on a ceramic or glass substrate. It was recognized, however, that there were considerable benefits to be had from producing an all-IC active filter.

Production of a quality on-chip capacitor involves substantial chip area, so the scaling techniques referred to earlier must be used to keep capacitance values down to the low picofarad range. The consequence of this is that, unfortunately, the circuit resistance values become proportionately large so that, again, there is a chip-area problem. The solution to this dilemma emerged in the late 1970s/early 1980s with the advent of the switched-capacitor (SC) active filter. This device, a development of the active-RC filter that is specifically intended for use in IC form, replaces prototype circuit resistors with arrangements of switches and capacitors that can be shown to simulate resistances, under certain circumstances. The great merit of the scheme is that the values of the capacitors involved in this process of resistor simulation are inversely proportional to the values of the prototype resistors; thus, the final IC structure involves principal and switched capacitors that are small in magnitude and hence ideal for IC realization. A good account of SC filters is given, for example, in Schaumann et al. [1990] and in Taylor and Huang [1997]. Commonly encountered techniques for SC filter design are based on the two major design styles (biquads and ladder simulation) that have been introduced in this section.

Many commercial IC active filters are based on SC techniques, and it is also becoming usual to find custom and semicustom IC design systems that include active filter modules as components within a macrocell library that the system-level design can simply invoke where analog filtering is required within an all-analog or mixed-signal analog/digital system.

Defining Terms

Active filter: An electronic filter whose design includes one or more active devices.

Biquad: An active filter whose transfer function comprises a ratio of second-order numerator and denominator polynomials in the frequency variable.

Electronic filter: An electronic circuit designed to transmit some range of signal frequencies while rejecting others. Phase and time-domain specifications may also occur.

Sensitivity: A measure of the extent to which a given circuit performance measure is affected by a given component within the circuit.

Related Topic

27.2 Applications

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Further Information

Tabulations of representative standard filter specification functions appear in the sources in the References by Schaumann et al. [1990] and Bowron and Stephenson [1979], but more extensive tabulations, including prototype passive filter component values, are given in A. I. Zverev, *Handbook of Filter Synthesis* (New York: John Wiley, 1967). More generally, the Schaumann text provides an admirable, up-to-date coverage of filter design with an extensive list of references as does Taylor and Huang [1997].

The field of active filter design remains active, and new developments appear in *IEEE Transactions on Circuits and Systems* and *IEE Proceedings Part G (Circuits and Systems).* The IEE publication *Electronic Letters* provides for short contributions. A number of international conferences (whose proceedings can be borrowed through technical libraries) feature active filter and related sessions, notably the *IEEE International Symposium on Circuits and Systems* (ISCAS) and the *European Conference on Circuit Theory and Design* (ECCTD).

29.2 Realization

J. W. Steadman and B. M. Wilamowski

After the appropriate low-pass form of a given **filter** has been synthesized, the designer must address the realization of the filter using **operational amplifiers**. If the required filter is not low-pass but high-pass, bandpass, or bandstop, transformation of the prototype function is also required [Budak, 1974; Van Valkenburg, 1982]. While a detailed treatment of the various transformations is beyond the scope of this work, most of the filter designs encountered in practice can be accomplished using the techniques given here.

When the desired filter function has been determined, the corresponding electronic circuit must be designed. Many different circuits can be used to realize any given transfer function. For purposes of this handbook, we present several of the most popular types of realizations. Much more detailed information on various circuit realizations and the advantages of each may be found in the literature, in particular Van Valkenburg [1982], Huelseman and Allen [1980], and Chen [1986]. Generally the design trade-offs in making the choice of circuit to be used for the realization involve considerations of the number of elements required, the sensitivity of the circuit to changes in component values, and the ease of tuning the circuit to given specifications. Accordingly, limited information is included about these characteristics of the example circuits in this section.

Each of the circuits described here is commonly used in the realization of **active filters**. When implemented as shown and used in the appropriate gain and bandwidth specifications of the amplifiers, they will provide excellent performance. Computer-aided filter design programs are available which simplify the process of obtaining proper element values and simulation of the resulting circuits [Krobe et al., 1989; Wilamowski et al., 1992].

Transformation from Low-Pass to Other Filter Types

To obtain a high-pass, bandpass, or bandstop filter function from a low-pass prototype, one of two general methods can be used. In one of these, the circuit is realized and then individual circuit elements are replaced by other elements or subcircuits. This method is more useful in **passive filter** designs and is not discussed further here. In the other approach, the transfer function of the low-pass prototype is transformed into the required form for the desired filter. Then a circuit is chosen to realize the new filter function. We give a brief description of the transformation in this section, then give examples of circuit realizations in the following sections.

Low-Pass to High-Pass Transformation

Suppose the desired filter is, for example, a high-pass Butterworth. Begin with the low-pass Butterworth transfer function of the desired order and then *transform* each pole of the original function using the formula

$$\frac{1}{S - S_j} \rightarrow \frac{Hs}{s - s_j} \tag{29.12}$$

which results in one complex pole and one zero at the origin for each pole in the original function. Similarly, each zero of the original function is transformed using the formula

$$S - S_j \rightarrow \frac{s - s_j}{Hs} \tag{29.13}$$

which results in one zero on the imaginary axis and one pole at the origin. In both equations, the scaling factors used are

$$H = \frac{1}{S_j} \quad \text{and} \quad s_j = \frac{\omega_0}{S_j} \tag{29.14}$$

where ω_0 is the desired cut-off frequency in radians per second.

Low-Pass to Bandpass Transformation

Begin with the low-pass prototype function in factored, or *pole-zero*, form. Then each pole is transformed using the formula

$$\frac{1}{S - S_j} \rightarrow \frac{Hs}{(s - s_1)(s - s_2)} \tag{29.15}$$

resulting in one zero at the origin and two conjugate poles. Each zero is transformed using the formula

$$S - S_j \rightarrow \frac{(s - s_1)(s - s_2)}{Hs} \tag{29.16}$$

resulting in one pole at origin and two conjugate zeros. In Eqs. (29.15) and (29.16)

$$H = -B; \quad s_{1,2} = \omega_c\left(\alpha \pm \sqrt{\alpha^2 - 1}\right); \text{ and } \alpha = \frac{BS_j}{2\omega_c} \tag{29.17}$$

where ω_c is the center frequency and B is the bandwidth of the bandpass function.

Low-Pass to Bandstop Transformation

Begin with the low-pass prototype function in factored, or pole-zero, form. Then each pole is transformed using the formula

$$\frac{1}{S - S_j} \rightarrow \frac{H(s - s_1)(s - s_2)}{(s - s_3)(s - s_4)} \tag{29.18}$$

transforming each pole into two zeros on the imaginary axis and into two conjugate poles. Similarly, each zero is transformed into two poles on the imaginary axis and into two conjugate zeros using the formula

$$S - S_j \rightarrow \frac{(s - s_3)(s - s_4)}{H(s - s_1)(s - s_2)} \tag{29.19}$$

where

$$H = \frac{1}{S_j}; \; s_{1,2} = \pm j\omega_c; \; s_{3,4} = \omega_c\left(\beta \pm \sqrt{\beta^2 - 1}\right); \text{ and } \beta = \frac{B}{2\omega_c S_j} \tag{29.20}$$

Once the desired transfer function has been obtained through obtaining the appropriate low-pass prototype and transformation, if necessary, to the associated high-pass, bandpass or bandstop function, all that remains is to obtain a circuit and the element values to realize the transfer function.

Circuit Realizations

Various electronic circuits can be found to implement any given transfer function. Cascade filters and ladder filters are two of the basic approaches for obtaining a practical circuit. Cascade realizations are much easier to find and to tune, but ladder filters are less sensitive to element variations. In cascade realizations, the transfer function is simply factored into first- and second-order parts. Circuits are built for the individual parts and then cascaded to produce the overall filter. For simple to moderately complex filter designs, this is the most common method, and the remainder of this section is devoted to several examples of the circuits used to obtain the first- and second-order filters. For very high-order transfer functions, ladder filters should be considered, and further information can be obtained by consulting the literature.

In order to simplify the circuit synthesis procedure, very often ω_0 is assumed to be equal to one and then after a circuit is found, the values of all capacitances in the circuit are divided by ω_0. In general, the following magnitude and frequency transformations are allowed:

$$R_{\text{new}} = K_M R_{\text{old}} \text{ and } C_{\text{new}} = \frac{1}{K_F K_M} C_{\text{old}} \tag{29.21}$$

where K_M and K_F are magnitude and frequency scaling factors, respectively.

Cascade filter designs require the transfer function to be expressed as a product of first- and second-order terms. For each of these terms a practical circuit can be implemented. Examples of these circuits are presented in Figs. 29.12–29.22. In general the following first- and second-order terms can be distinguished:

(a) First-order low-pass:

$$T(s) = \frac{H\omega_0}{s + \omega_0}$$

Assumption : $r_1 = 1$

$$c_1 = \frac{1}{\omega_0} \qquad r_2 = |H|\,\omega_0$$

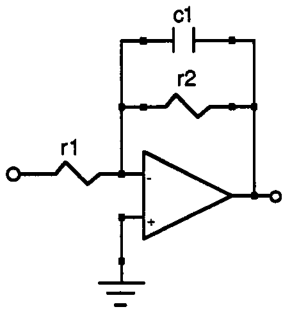


FIGURE 29.12 First-order low-pass filter.

This filter is inverting, i.e., H must be negative, and the scaling factors shown in Eq. (29.21) should be used to obtain reasonable values for the components.

(b) First-order high-pass:

$$T(s) = \frac{Hs}{s + \omega_0}$$

Assumption : $r_1 = 1$

$$c_1 = \frac{1}{\omega_0} \qquad r_2 = |H|$$

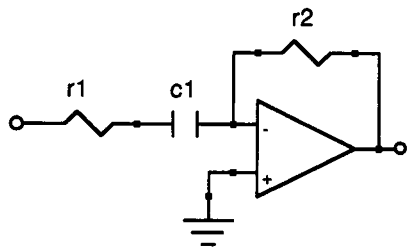


FIGURE 29.13 First-order high-pass filter.

This filter is inverting, i.e., H must be negative, and the scaling factors shown in Eq. (29.21) should be used to obtain reasonable values for the components.

While several passive realizations of first-order filters are possible (low-pass, high-pass, and lead-lag), the active circuits shown here are inexpensive and avoid any loading of the other filter sections when the individual circuits are cascaded. Consequently, these circuits are preferred unless there is some reason to avoid the use of the additional operational amplifier. Note that a second-order filter can be realized using one operational amplifer as shown in the following paragraphs, so it is common practice to choose even-order transfer functions, thus avoiding the use of any first-order filters.

(c) There are several second-order low-pass circuits:

$$T(s) = \frac{H\omega_0^2}{s^2 + \frac{\omega_0}{Q}s + \omega_0^2}$$

Assumption : $r_1 = r_2 = 1$

$$c_1 = \frac{2Q}{\omega_0} \qquad c_2 = \frac{1}{2Q\omega_0}$$

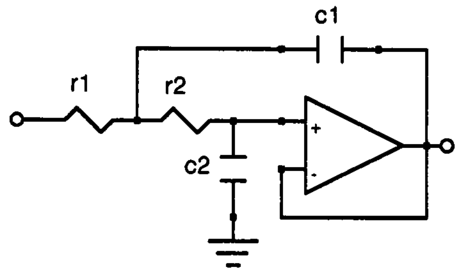


FIGURE 29.14 Second-order low-pass Sallen-Key filter.

This filter is noninverting and unity gain, i.e., H must be one, and the scaling factors shown in Eq. (29.21) should be used to obtain reasonable element values. This is a very popular filter for realizing second-order functions because it uses a minimum number of components and since the operation amplifier is in the unity gain configuration it has very good bandwidth.

Another useful configuration for second-order low-pass filters uses the operational amplifier in its inverting "infinite gain" mode as shown in Fig. 29.15.

$$T(s) = \frac{H\omega_0^2}{s^2 + \frac{\omega_0}{Q}s + \omega_0^2}$$

Assumption : $r_1 = r_2 = r_3 = 1$

$$c_1 = \frac{3Q}{\omega_0} \qquad c_2 = \frac{1}{3Q\omega_0}$$

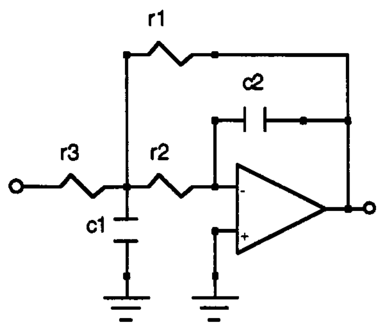


FIGURE 29.15 Second-order low-pass filter using the inverting circuit.

This circuit has the advantage of relatively low sensitivity of ω_0 and Q to variations in component values. In this configuration the operational amplifier's gain-bandwidth product may become a limitation for high-Q and high-frequency applications [Budak, 1974]. There are several other circuit configurations for low-pass filters. The references given at the end of the section will guide the designer to alternatives and the advantages of each.

(d) Second-order high-pass filters may be designed using circuits very much like those shown for the low-pass realizations. For example, the Sallen-Key low-pass filter is shown in Fig. 29.16.

$$T(s) = \frac{Hs^2}{s^2 + \frac{\omega_0}{Q}s + \omega_0^2}$$

Assumption : $r_3 = 1$

$$c_1 = c_2 = 1$$

$$r_1 = r_2 = \frac{1}{\omega_0} \qquad r_4 = 2 - \frac{1}{Q}$$

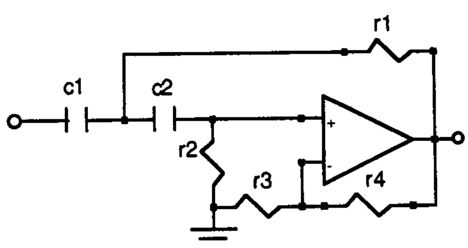


FIGURE 29.16 A second-order high-pass Sallen-Key filter.

As in the case of the low-pass Sallen-Key filter, this circuit is noninverting and requires very little gain from the operational amplifier. For low to moderate values of Q, the **sensitivity functions** are reasonable and the circuit performs well.

The inverting *infinite gain* high-pass circuit is shown in Fig. 29.17 and is similar to the corresponding low-pass circuit.

$$T(s) = \frac{Hs^2}{s^2 + \frac{\omega_0}{Q}s + \omega_0^2}$$

$$\text{Assumption: } r_1 = 1$$

$$r_2 = 9Q^2 \quad c_1 = c_2 = c_3 = \frac{1}{3Q^2}$$

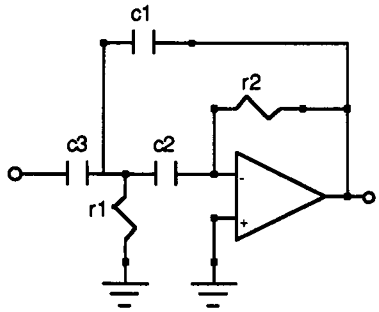


FIGURE 29.17 An inverting second-order high-pass circuit.

This circuit has relatively good sensitivity figures. The principal limitation occurs with high-Q filters since this requires a wide spread of resistor values.

Both low-pass and high-pass frequency response circuits can be achieved using three operational amplifier circuits. Such circuits have some sensitivity function and tuning advantages but require far more components. These circuits are used in the sections describing bandpass and bandstop filters. The designer wanting to use the three-operational-amplifier realization for low-pass or high-pass filters can easily do this using simple modifications of the circuits shown in the following sections.

(e) Second-order bandpass circuits may be realized using only one operational amplifier. The Sallen-Key filter shown in Fig. 29.18 is one such circuit.

$$T(s) = \frac{H\frac{\omega_0}{Q}s}{s^2 + \frac{\omega_0}{Q}s + \omega_0^2}$$

$$\text{Assumption: } c_1 = c_2 = 1; \quad r_5 = 1$$

$$r_2 = r_3 = \frac{\sqrt{2}}{\omega_0} \qquad r_1 = \frac{\frac{4Q}{\sqrt{2}} - 1}{H}$$

$$r_4 = \frac{\frac{4Q}{\sqrt{2}} - 1}{\frac{4Q}{\sqrt{2}} - 1 - H} \qquad r_6 = 3 - \frac{\sqrt{2}}{\omega_0}$$

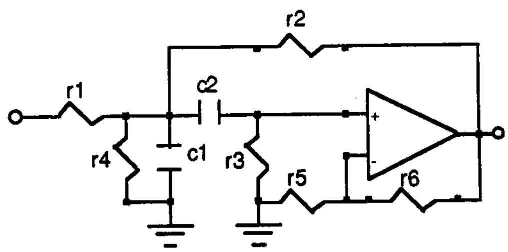


FIGURE 29.18 A Sallen-Key bandpass filter.

This is a noninverting amplifier which works well for low- to moderate-Q filters and is easily tuned [Budak, 1974]. For high-Q filters the sensitivity of Q to element values becomes high, and alternative circuits are recommended. One of these is the bandpass version of the inverting amplifier filter as shown in Fig. 29.19.

$$T(s) = \frac{H \frac{\omega_0}{Q} s}{s^2 + \frac{\omega_0}{Q} s + \omega_0^2}$$

$$\text{Assumption : } c_1 = c_2 = \frac{1}{2Q\omega_0}$$

$$r_1 = \frac{2Q^2}{H} \qquad r_2 = 4Q^2 \qquad r_3 = \frac{1}{1 - \frac{H}{2Q^2}}$$

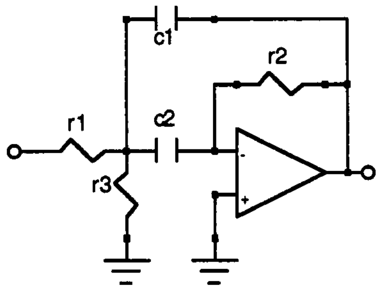
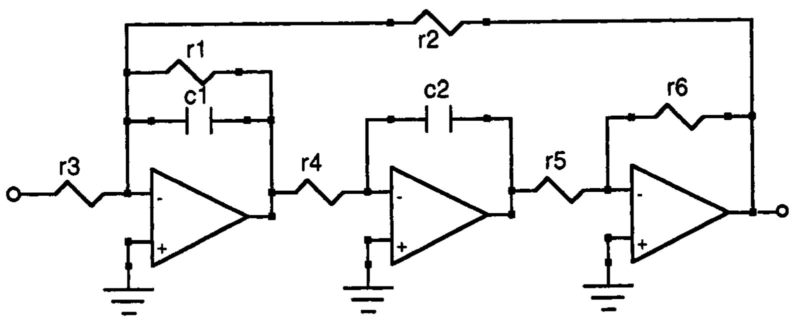


FIGURE 29.19 The inverting amplifier bandpass filter.

This circuit has few components and relatively small sensitivity of ω_0 and Q to variations in element values. For high-Q circuits, the range of resistor values is quite large as r_1 and r_2 are much larger than r_3.

When ease of tuning and small sensitivities are more important than the circuit complexity, the three-operational-amplifier circuit of Fig. 29.20 may be used to implement the bandpass transfer function.



$$T(s) = \frac{H \frac{\omega_0}{Q} s}{s^2 + \frac{\omega_0}{Q} s + \omega_0^2} \qquad c_1 = c_2 = \frac{1}{\omega_0} \qquad r_1 = Q \qquad r_2 = r_4 = r_5 = r_6 = 1 \qquad r_3 = \frac{Q}{|H|}$$

FIGURE 29.20 The three-operational-amplifier bandpass filter.

The filter as shown in Fig. 29.20 is inverting. For a noninverting realization, simply take the output from the middle amplifier rather than the right one. This same configuration can be used for a three-operational-amplifier low-pass filter by putting the input into the summing junction of the middle amplifier and taking the output from the left operational amplifier. Note that Q may be changed in this circuit by varying r_1 and that this will not alter ω_0. Similarly, ω_0 can be adjusted by varying c_1 or c_2 and this will not change Q. If only variable resistors are to be used, the filter can be tuned by setting ω_0 using any of the resistors other than r_1 and then setting Q using r_1.

(f) Second-order bandstop filters are very useful in rejecting unwanted signals such as line noise or carrier frequencies in instrumentation applications. Such filters are implemented with methods very similar to the bandpass filters just discussed. In most cases, the frequency of the zeros is to be the same as the frequency of the poles. For this application, the circuit shown in Fig. 29.21 can be used.

$$T(s) = \frac{H(s^2 + \omega_z^2)}{s^2 + \frac{\omega_0}{Q}s + \omega_0^2}$$

Assumption : $c_1 = c_2 = 1$

$$r_1 = \frac{1}{2Q\omega_0} \qquad r_3 = \frac{1}{Q\omega_0} \qquad r_2 = r_4 = \frac{2Q}{\omega_0}$$

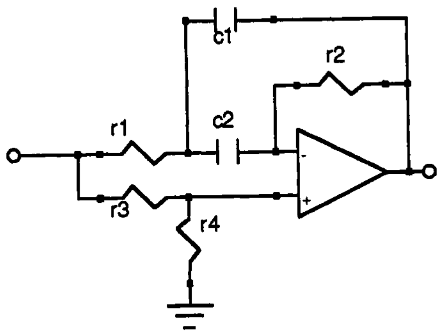
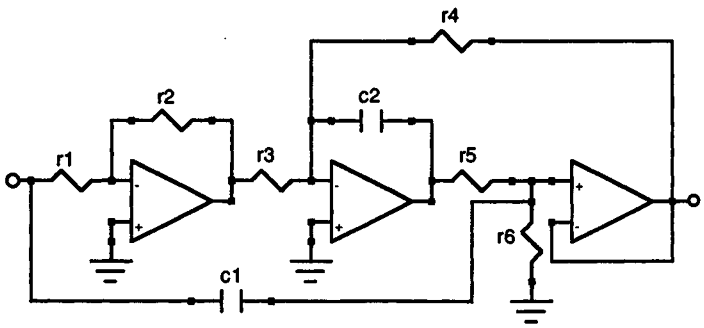


FIGURE 29.21 A single operational-amplifier bandstop filter.

The primary advantage of this circuit is that it requires a minimum number of components. For applications where no tuning is required and the Q is low, this circuit works very well. When the bandstop filter must be tuned, the three-operational-amplifier circuit is preferable.



$$T(s) = \frac{H(s^2 + \omega_z^2)}{s^2 + \frac{\omega_0}{Q}s + \omega_0^2} \quad c_1 = c_2 = \frac{1}{\omega_0} \quad r_1 = 1 \quad r_2 = H \quad r_5 = r_6 = 2Q \quad r_3 = \frac{H\omega_0^2}{2Q\omega_z^2} \quad r_4 = \frac{1}{2Q}$$

FIGURE 29.22 A three-operational-amplifier bandstop filter.

The foregoing circuits provide a variety of useful first- and second-order filters. For higher-order filters, these sections are simply cascaded to realize the overall transfer function desired. Additional detail about these circuits as well as other circuits used for active filters may be found in the references.

Defining Terms

Active filter: A filter circuit which uses active components, usually operational amplifiers.

Filter: A circuit which is designed to be frequency selective. That is, the circuit will emphasize or "pass" certain frequencies and attenuate or "stop" others.

Operational amplifier: A very high-gain differential amplifier used in active filter circuits and many other applications. These monolithic integrated circuits typically have such high gain, high input impedance, and low output impedance that they can be considered "ideal" when used in active filters.

Passive filter: Afilter circuit which uses only passive components, i.e., resistors, inductors, and capacitors. These circuits are useful at higher frequencies and as prototypes for ladder filters that are active.

Sensitivity function: A measure of the fractional change in some circuit characteristic, such as center frequency, to variations in a circuit parameter, such as the value of a resistor. The sensitivity function is normally defined as the partial derivative of the desired circuit characteristic with respect to the element value and is usually evaluated at the nominal value of all elements.

Related Topics

10.3 The Ideal Linear-Phase Low-Pass Filter • 27.1 Ideal and Practical Models

References

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L.P. Huelseman and P.E. Allen, *Introduction to the Theory and Design of Active Filters*, New York: McGraw-Hill, 1980.

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M.E. Van Valkenburg, *Analog Filter Design*, New York: Holt, Rinehart and Winston, 1982.

B.M. Wilamowski, S.F. Legowski, and J.W. Steadman, "Personal computer support for teaching analog filter analysis and design," *IEEE Trans. on Education*, vol. 35, no. 4, November 1992.

Further Information

The monthly journal *IEEETransactions on Circuits and Systems* is one of the best sources of information on new active filter functions and associated circuits.

The British journal *Electronics Letters* also often publishes articles about active circuits.

The *IEEETransactions on Education* has carried articles on innovative approaches to active filter synthesis as well as computer programs for assisting in the design of active filters.

29.3 Generalized Impedance Convertors and Simulated Impedances

James A. Svoboda

The problem of designing a circuit to have a given transfer function is called filter design. This problem can be solved using passive circuits, that is, circuits consisting entirely of resistors, capacitors, and inductors. Further, these passive filter circuits can be designed to have some attractive properties. In particular, passive filters can be designed so that the transfer function is relatively insensitive to variations in the values of the resistances, capacitances, and inductances. Unfortunately, passive circuits contain inductors. Inductors are frequently large, heavy, expensive, and nonlinear.

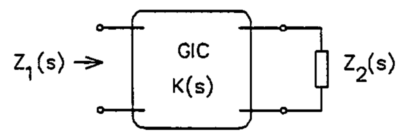

FIGURE 29.23 The GIC converts the impedance $Z_2(s)$ to the impedance $Z_1(s)$.

Generalized impedance convertors (GIC) are electronic circuits used to convert one impedance into another impedance [Bruton, 1981; Van Valkenburg, 1982]. GICs provide a way to get the advantages of passive circuits without the disadvantages of inductors. Figure 29.23 illustrates the application of a GIC. The GIC converts the impedance $Z_2(s)$ to the impedance $Z_1(s)$. The impedances are related by

$$Z_1(s) = K(s)Z_2(s) \tag{29.22}$$

The function $K(s)$ is called the conversion function or, more simply, the gain of the GIC.

Figure 29.24 shows two ways to implement a GIC using operational amplifiers (op amps). The GIC shown in Fig. 29.24a has a gain given by

$$K(s) = -\frac{Z_A(s)}{Z_B(s)} \tag{29.23}$$

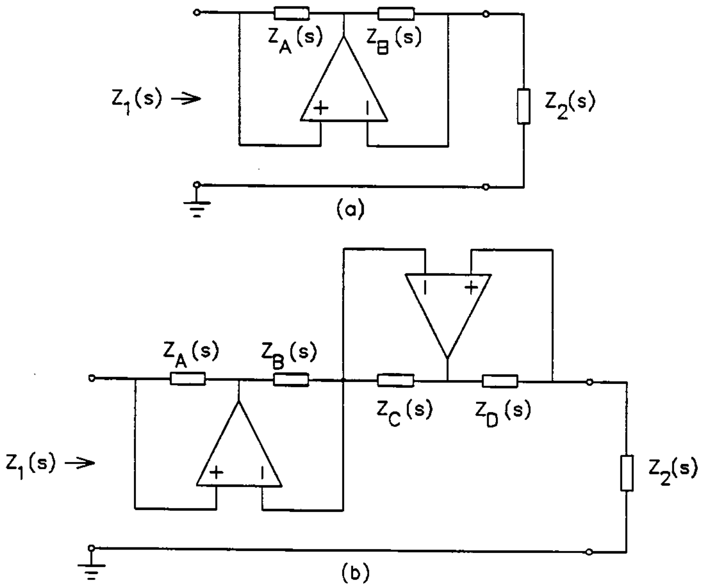

FIGURE 29.24 (a) An inverting GIC and (b) a noninverting GIC.

This GIC is called an inverting GIC because $K(s)$ is negative.

A **negative resistor** is an electronic circuit that acts like a resistor having a negative value of resistance. The inverting GIC can be used to design a negative resistor by taking $Z_A(s) = R_A$, $Z_B(s) = R_B$, and $Z_2(s) = R_2$. Figure 29.25(a) shows the op amp circuit that implements a negative resistor, and Fig. 29.25(b) shows the equivalent circuit. The resistance of the negative resistor is given by

$$R = -\frac{R_A}{R_B} R_2 \tag{29.24}$$

Figure 29.24(b) shows another op amp circuit that implements a GIC. The gain of this GIC is given by

$$K(s) = \frac{Z_A(s)Z_C(s)}{Z_B(s)Z_D(s)} \tag{29.25}$$

This GIC is called a noninverting GIC because $K(s)$ is positive.

A **simulated inductor** is circuit consisting of resistors, capacitors, and amplifiers that acts like an inductor. The noninverting GIC can be used to design a simulated inductor by taking $Z_A(s) = R_A$, $Z_B(s) = R_B$, $Z_C(s) = R_C$, $Z_D(s) = 1/(sC_D)$, and $Z_2(s) = R_2$. Figure 29.25(c) shows the op amp circuit that implements a simulated inductor, and Fig. 29.25(d) shows the equivalent circuit. The inductance of the simulated inductor is given by

$$L = \frac{R_A R_C C_D}{R_B} R_2 \tag{29.26}$$

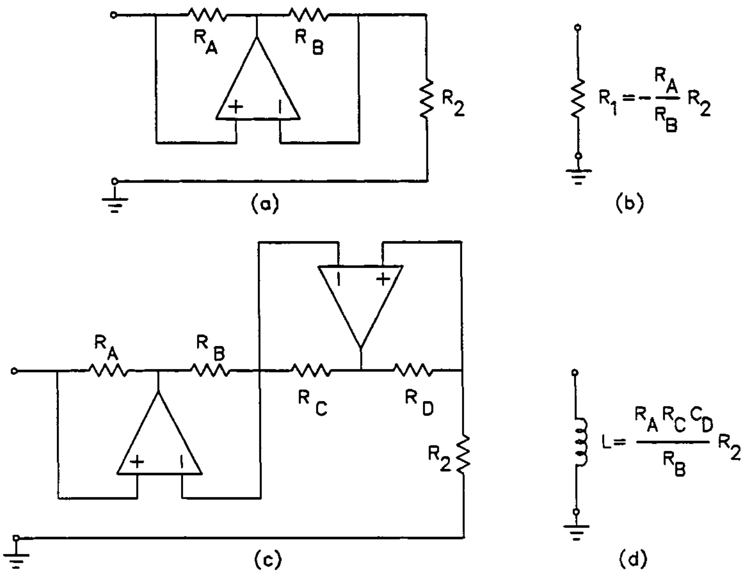

FIGURE 29.25 (a) A grounded negative resistor and (b) its equivalent circuit. (c) A grounded simulated inductor and (d) its equivalent circuit.

Notice that one node of the negative resistor shown in Fig. 29.25(b) and of the simulated inductor shown in Fig. 29.25(d) is grounded. This ground is the ground of the power supplies used to bias the op amp. Op amp circuits implementing floating negative resistors and simulated inductors are more difficult to design [Reddy, 1976]. Floating negative resistors and simulated inductors can be more easily designed using an electronic device called a **current conveyor**. The symbol for the current conveyor is shown in Fig. 29.26. The terminal voltages and currents of the "second-generation" current conveyor [Sedra and Smith, 1971] are represented by

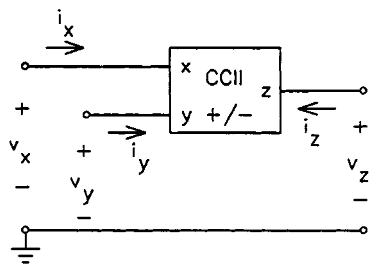

FIGURE 29.26 A CCII current conveyor.

$$\begin{pmatrix} i_y \\ v_x \\ i_z \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & \pm 1 & 0 \end{pmatrix} \begin{pmatrix} v_y \\ i_x \\ v_z \end{pmatrix} \tag{29.27}$$

There are two kinds of second-generation current conveyor, corresponding to the two possible signs of the ±1 entry in the third row of Eq. (29.27). The + indicates a $CCII^+$ current conveyor while the – indicates a $CCII^-$ current conveyor.

Current conveyors are related to **transimpedance amplifiers** [Svoboda, 1991]. Figure 29.27(a) indicates that a transimpedance amplifier consists of a $CCII^+$ current conveyor and a voltage buffer. Several transimpedance amplifiers, e.g., the AD844, AD846, and AD811, are commercially available. Figure 29.27(b) shows that a $CCII^-$ current conveyor can be constructed from two $CCII^+$ current conveyors.

Figure 29.28(a) presents a current conveyor circuit that implements a floating negative resistor. The resistance of the negative resistor is given simply as

$$R = -R_2 \tag{29.28}$$

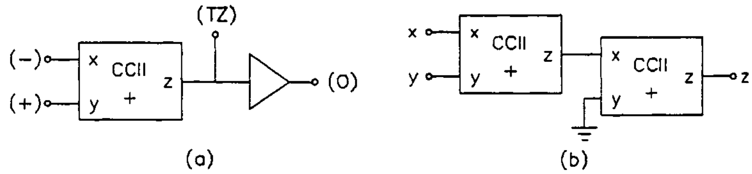

FIGURE 29.27 (a) A transimpedance amplifier consists of a CCII$^+$ current conveyor and a voltage buffer. (b) A CCII$^-$ implemented using two CCII$^+$ current conveyors.

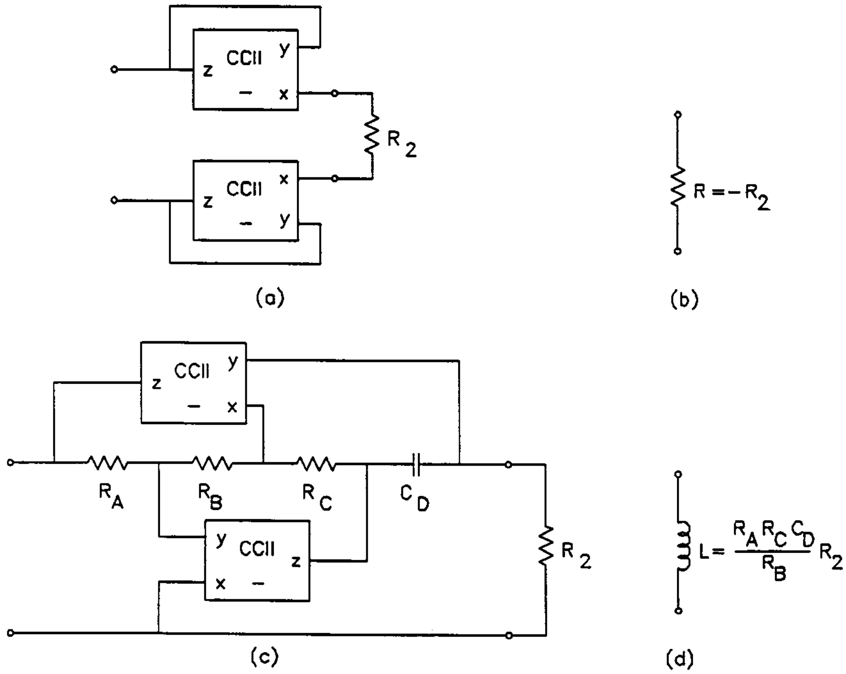

FIGURE 29.28 (a) A floating negative resistor and (b) its equivalent circuit. (c) A floating simulated inductor and (d) its equivalent circuit.

Figure 29.28(b) shows the equivalent circuit of the current conveyor negative resistor. Notice that in Fig. 29.28(b) neither node is required to be ground, in contrast to the equivalent circuit for the op amp negative resistor in Fig. 29.25(b).

Figure 29.28(c) shows a current conveyor circuit that implements a floating simulated inductor. The inductance of this simulated inductor is given by

$$L = \frac{R_A R_C C_D}{R_B} R_2 \tag{29.29}$$

Figure 29.28(d) shows the equivalent circuit of the current conveyor simulated inductor. The current conveyor circuit can simulate a floating inductor, so neither node of the equivalent inductor is required to be grounded.

Figure 29.29 illustrates an application of simulated impedances. The circuit shown in Fig. 29.29(a) implements a voltage-controlled current source (VCCS). This particular VCCS has the advantage of perfect regulation. In other words, the output current, i_o, is completely independent of the load resistance, R_L. The circuit in Fig. 29.29(a) requires a negative resistor, the resistor labeled $-R$. Since one node of this resistor is grounded,

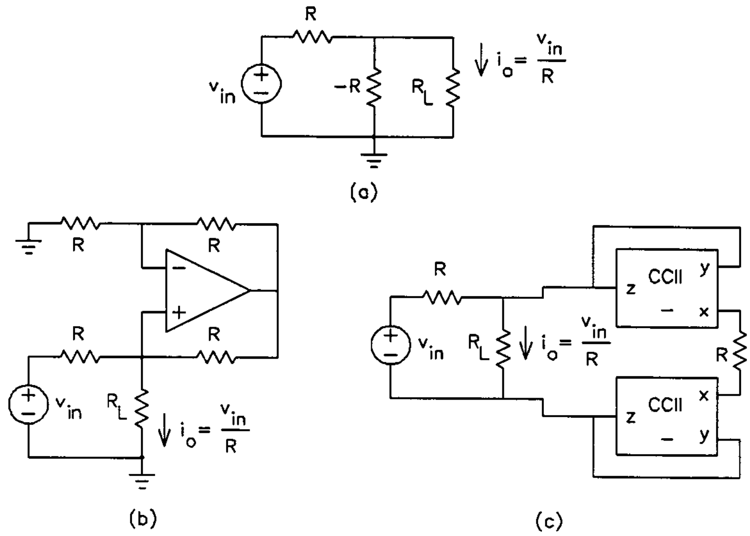

FIGURE 29.29 Three versions of a VCCS: (a) using a negative resistor, (b) using an op amp, and (c) using current conveyors.

this resistor can be implemented using the op amp negative resistor shown in Fig. 29.25(a). The resulting circuit is shown in Fig. 29.29(b).

In Fig. 29.29(a), one node of the load resistor is grounded. As a consequence, one node of the negative resistor was grounded and it was appropriate to use the op amp negative resistor. Sometimes a VCCS is needed to cause a current in an ungrounded load resistance. In this case the negative resistor must also be ungrounded so the current conveyor negative resistor is used. In Fig. 29.29(c) the current conveyor negative resistor is used to implement a VCCS that supplies current to an ungrounded resistor R_L.

Figure 29.30 illustrates the application of a simulated inductor. The circuit shown in Fig. 29.30(a) is a low-pass filter. The transfer function of this filter is

$$\frac{V_o(s)}{V_{\text{in}}(s)} = \frac{\frac{1}{LC}}{s^2 + \frac{R}{L}s + \frac{1}{LC}} \tag{29.30}$$

The filter in Fig. 29.30(a) contains an inductor. This inductor can be implemented as a simulated inductor. Since neither node of the inductor is grounded, it is necessary to use the current conveyor simulated inductor. The resulting circuit is shown in Fig. 29.30(b). The inductance of the simulated inductor is given by Eq. (29.29). Substituting this equation into Eq. (29.30) gives the transfer function of the circuit in Fig. 29.30(b)

$$\frac{V_o(s)}{V_{\text{in}}(s)} = \frac{\frac{R_B}{R_A R_C R_2 C_D C}}{s^2 + \frac{R R_B}{R_A R_C R_2 C_D}s + \frac{R_B}{R_A R_C R_2 C_D C}} \tag{29.31}$$

Similarly, high-pass, bandpass, and notch filters can be designed by rearranging the resistor, capacitor, and inductor in Fig. 29.30(a) to get the desired transfer function and then simulating the inductor. When the inductor is grounded, it can be simulated using the op amp–simulated inductor, but when the inductor is floating, the current conveyor–simulated inductor must be used.

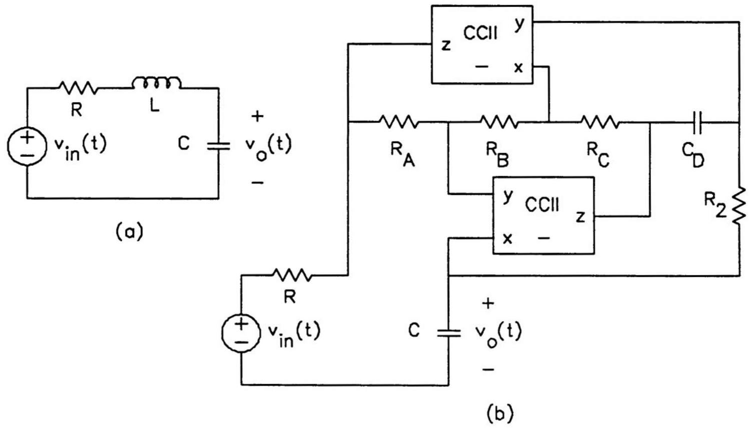

FIGURE 29.30 (a) A low-pass RLC filter and (b) the same low-pass filter implemented using a floating simulated inductor.

Defining Terms

Current conveyor: An electronic device represented by Fig. 29.26 and Eq. (29.27).

Generalized impedance convertors (GIC): Electronic circuits used to convert one impedance into another impedance.

Negative resistor: An electronic circuit that acts like a resistor having a negative value of resistance.

Transimpedance amplifier: An amplifier consisting of a $CCII^+$ current conveyor and a voltage buffer.

Simulated inductor: A circuit consisting of resistors capacitors and amplifiers that acts like an inductor.

Related Topic

27.1 Ideal and Practical Models

References

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Further Information

Additional information regarding current conveyors can be found in *Analogue IC Design: The Current Mode Approach* edited by Toumazou, Lidgey, and Haigh. *The Circuits and Filters Handbook* edited by Wai-Kai Chen provides background on circuit design in general and on filters in particular. Several journals, including *IEEE Transactions on Circuits and Systems, The International Journal of Electronics,* and *Electronic Letters,* report on advances in filter design.