

Second-order Active Filter(Biquad)

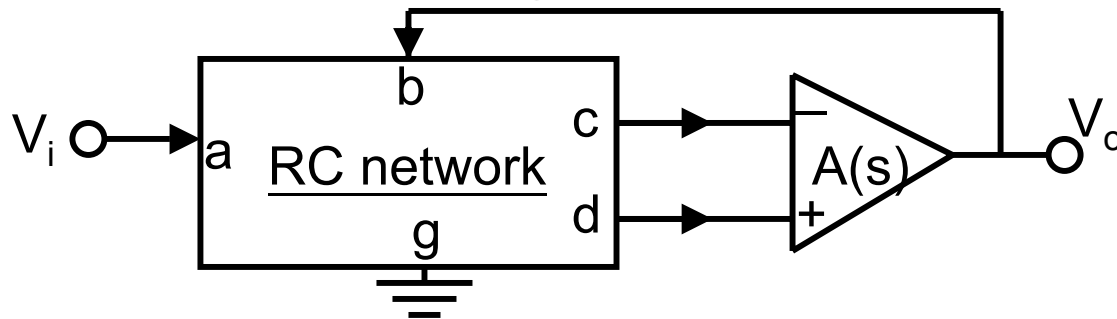
- Biquad transfer function

$$H(s) = \frac{a_2s^2 + a_1s + a_0}{s^2 + b_1s + b_0} = \frac{a_2(s^2 + (w_z/Q_z)s + w_z^2)}{s^2 + (w_p/Q_p)s + w_p^2}$$

- Biquads
 - fundamental for high-order cascade or multiple loop feedback filters
 - best possible
 - a, low sensitivity
 - b, economical
 - c, manufacturable
 - d, easy to tune
 - e, low output resistance (for voltage output)
 - f, high input impedance
 - different circuits are claimed by their respective inventors to be advantageous in one way or another

Single-Amplifier Biquad (SAB)

- General feedback configuration



- transfer function of RC network

$$T_{kl}(s) = \frac{V_k}{V_l} = \frac{N_{kl}(s)}{D_o(s)} \quad \text{Where } k=c,d \quad l=a,b$$

$$\frac{1}{A(s)} V_o(s) = (T_{da}(s) - T_{ca}(s)) V_i(s) + (T_{db}(s) - T_{cb}(s)) V_o(s)$$

$$H(s) = \frac{V_o(s)}{V_i(s)} = \frac{T_{da}(s) - T_{ca}(s)}{T_{cb}(s) - T_{db}(s) + 1/A(s)}$$

- Second-order RC network is required for 2nd-order $H(s)$
- Transmission zeros are determined by the forward path, i.e., $T_{ca}(s)$ and $T_{da}(s)$
- The poles are set by the feedback paths, i.e., $T_{cb}(s)$ and $T_{db}(s)$

Generation of transfer function zeros

- Natural frequencies or poles are found from

$$T_{cb}(s) - T_{db}(s) + 1/A(s) = 0$$

=> Poles does not depend on node a, I.e., it is independent of the connection of input V_i .

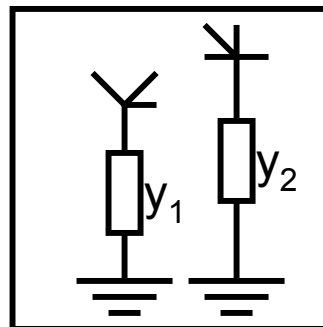
- Zeros of $H(s)$ are found from

$$T_{da}(s) - T_{ca}(s) = 0 \quad (\text{depends on node a})$$

=> Zeros depend on where the input signal is applied.

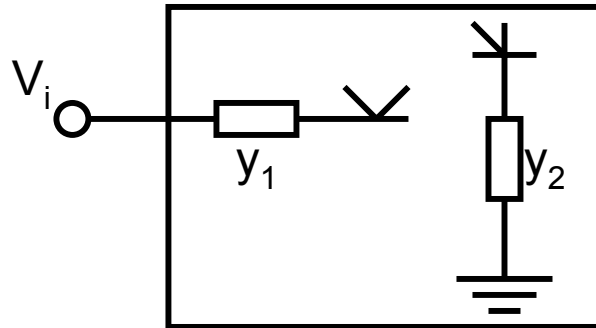
=> The zeros of a transfer function are created without disturbing the poles, by feeding the input signal into any node or nodes that were previously connected to ground.

- Example : by feeding input signal into one node
 - Two ground elements

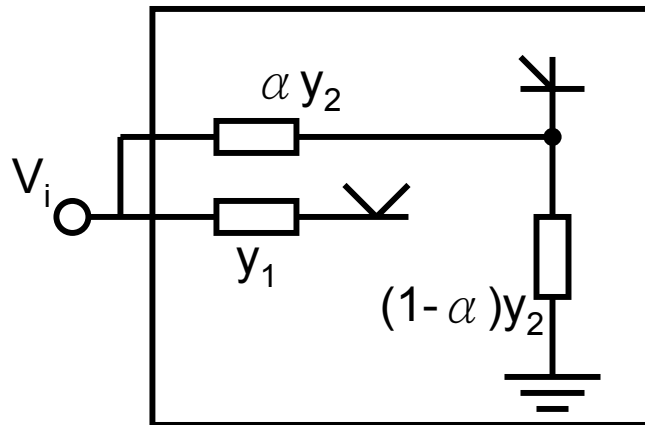


Generation of transfer function zeros(cont.)

- One element is completely lifted off ground and connected to input.

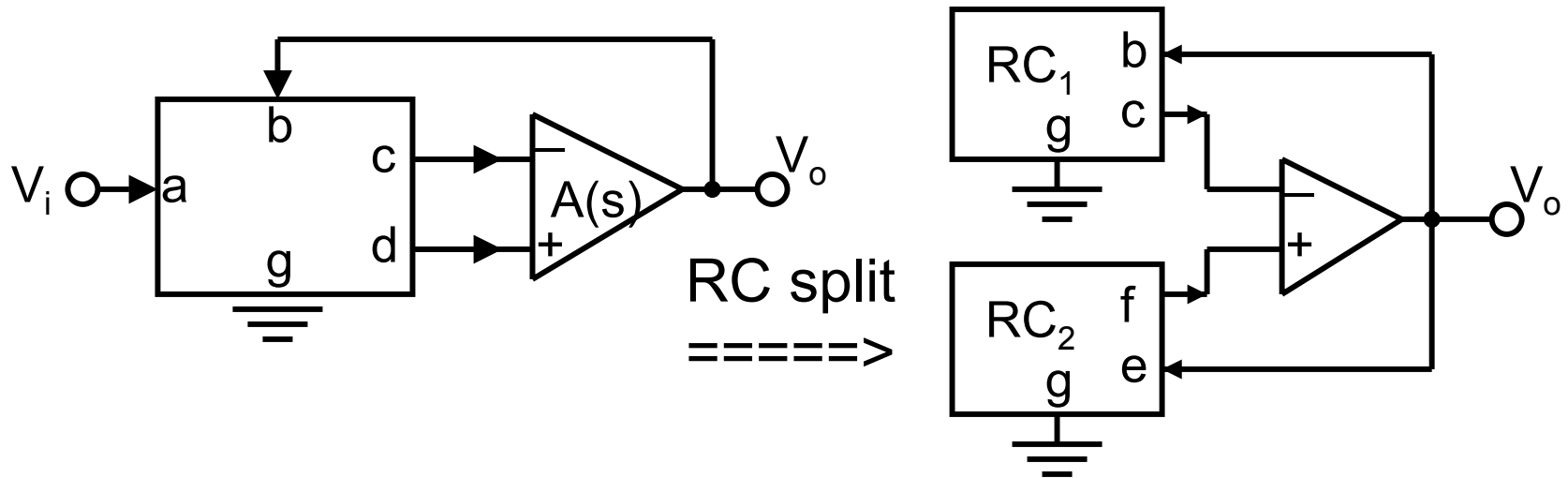


- In addition, the other element is partially lifted off ground and connected to input.



Generation of transfer function poles

- RC network is split into two parts (input node a is ignored)



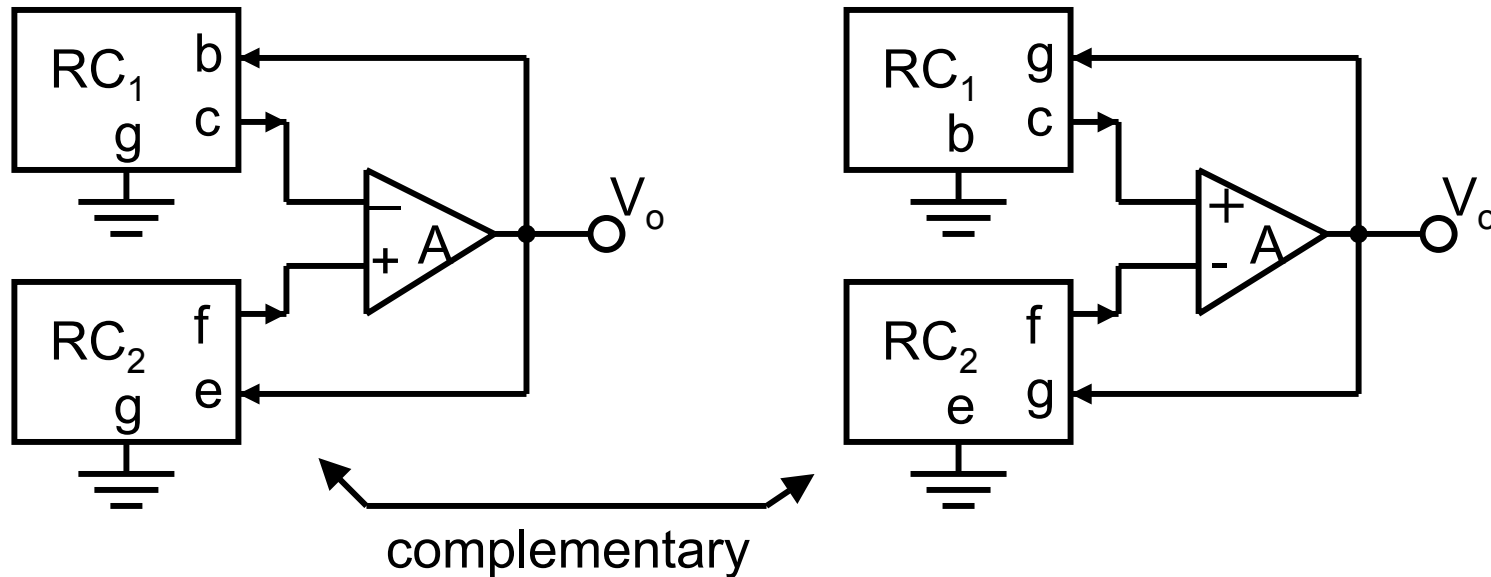
Poles determined by
 $T_{cb}(s) - T_{db}(s) + 1/A(s) = 0$

Poles determined by
 $T_{cb}(s) - T_{fe}(s) + 1/A(s) = 0$

- Complementary network
 - interchanges 1. two OPAMP input terminals each other
 - 2. OPAMP output with ground
 - natural frequency is the same as that of the original network

Generation of transfer function poles(cont.)

– Example(V_i ignored)



Poles are determined by

$$T_{cb}(s) - T_{fe}(s) + 1/A(s) = 0$$

Poles are determined by

$$T_{fg}(s) - T_{cg}(s) + 1/A(s) = 0$$

$$\left\{ \begin{aligned} T_{fg}(s) &= \frac{V_{fe}(s)}{V_{ge}(s)} = \frac{V_{fg}(s) + V_{ge}(s)}{V_{ge}(s)} = 1 + \frac{V_{fg}(s)}{V_{ge}(s)} = 1 - \frac{V_{fg}(s)}{V_{eg}(s)} = 1 - T_{fe}(s) \quad \text{----- (A)} \\ T_{cg}(s) &= \frac{V_{cb}(s)}{V_{gb}(s)} = \frac{V_{cg}(s) + V_{gb}(s)}{V_{gb}(s)} = 1 + \frac{V_{cg}(s)}{V_{gb}(s)} = 1 - \frac{V_{cg}(s)}{V_{bg}(s)} = 1 - T_{cb}(s) \quad \text{----- (B)} \end{aligned} \right.$$

Generation of transfer function poles(cont.)

$$\Rightarrow T_{fg}(s) - T_{cg}(s) + 1/A(s) = 1 - T_{fe}(s) - (1 - T_{cb}(s)) + 1/A(s) = T_{cb}(s) - T_{fe}(s) + 1/A(s)$$

\Rightarrow The above two systems are complementary

- From Eqs. (A) and (B)

\Rightarrow Input and ground terminals in a network are complements

\Rightarrow If the transfer function from input to output is $H(s)$, then interchanging input and ground connections gives the transfer function $H_c(s) = 1 - H(s)$

– Examples:

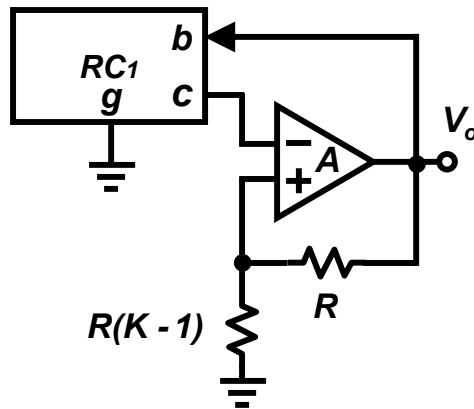
(1) If we have a bandpass circuit realizing $H_{BP}(s) = \frac{as}{s^2 + as + b}$, then interchanging input and ground results in a notch filter $H_N(s) = 1 - H_{BP}(s) = \frac{s^2 + b}{s^2 + as + b}$

(2) If $H_{BP}(s) = \frac{2as}{s^2 + as + b}$, then $1 - H_{BP}(s) = H_{AP}(s) = \frac{s^2 - as + b}{s^2 + as + b}$

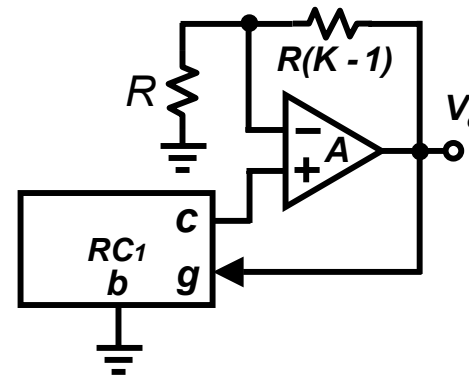
Enhanced Feedback Structure

- Enhanced negative feedback (ENF) and enhanced positive feedback (EPF) (They both are complementary each other)
 - structure (input not shown)

1. ENF



2. EPF



- Positive feedback in ENF results in Q enhancement.

Its complementary structure, EPF, also has Q enhancement.

Enhanced Feedback Structure (Cont.)

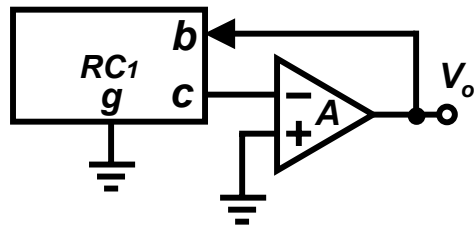
– Their pole positions are given by

$$\underbrace{T_{cb}(s) - \frac{K-1}{K} + \frac{1}{A(s)}}_{\text{ENF}} = 1 - \underbrace{T_{cg}(s) - \frac{K-1}{K} + \frac{1}{A(s)}}_{\text{EPF}} = \frac{1}{K} - T_{cg}(s) + \frac{1}{A(s)} = 0$$

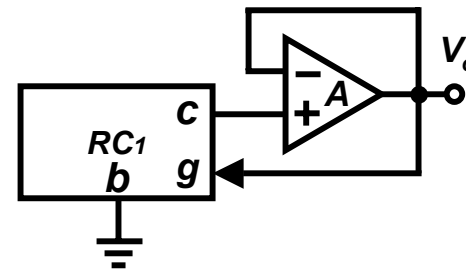
- Special cases : Infinite-gain negative-feedback (NF) and unity-gain positive-feedback (PF)

– structure (input not shown)

1. NF



2. PF



Enhanced Feedback Structure (Cont.)

- NF is a special case of ENF
Its complementary structure, PF, is a special case of EPF.
- Dependence of pole positions of the ENF and the EPF networks on circuit element is identical.
- For NF&PF, pole sensitivity is the same for designing filters with either unity-gain or infinite-gain OPAMP.
- Their pole positions are given by

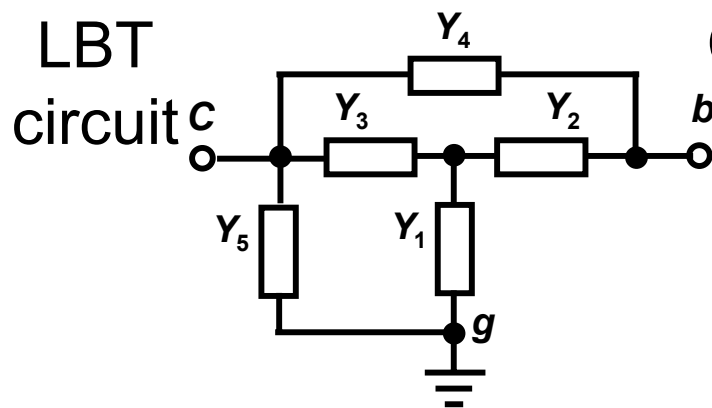
$$\underbrace{T_{cb}(s) + \frac{1}{A(s)}}_{\text{NF}} = \underbrace{1 - T_{cg}(s) + \frac{1}{A(s)}}_{\text{PF}} = 0$$

Passive RC Twoports for Single-Amplifier Biquad

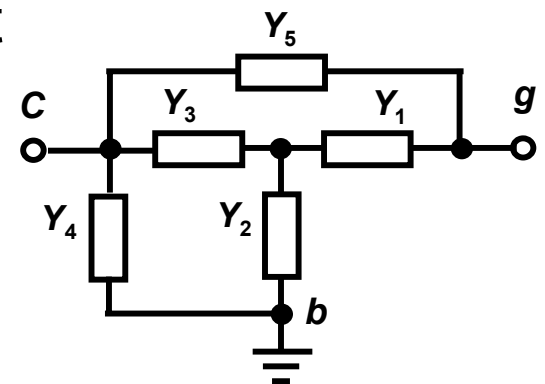
- RC network of feedback filter structure, ENF or EPF, should be used to realize transfer functions $T_{cb}(s)$ or $T_{cg}(s)$ of the form of

$$T(s) = \frac{a_2 s^2 + a_1 s + a_0}{s^2 + s \frac{w_1}{Q_p} + w_1^2} \quad (\text{for poles generation of single-amplifier biquad})$$

- Loaded bridged T (LBT)
 - complementary to itself
 - pole generation

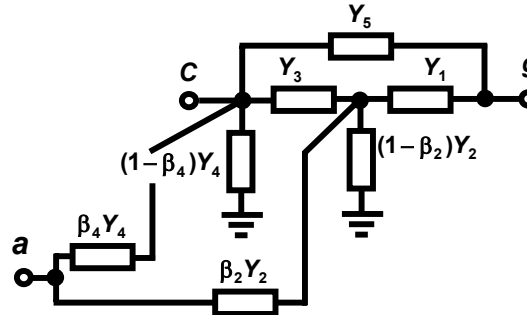
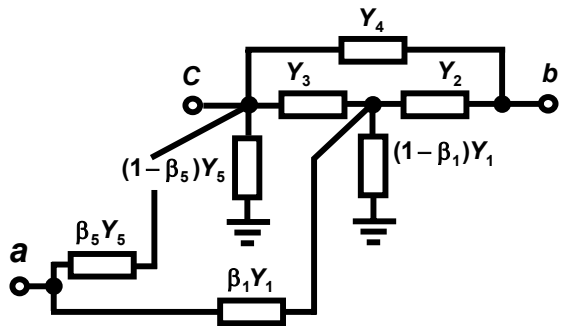


Complement of LBT



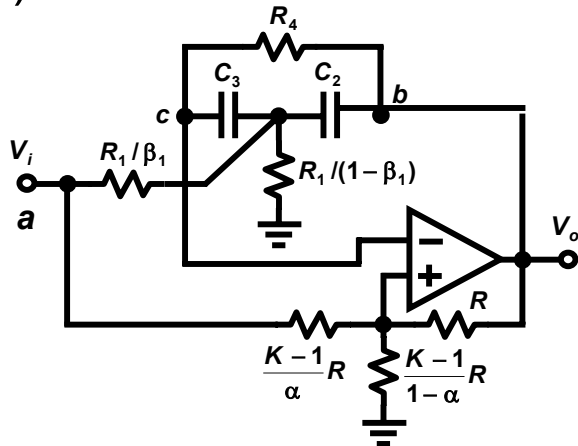
Passive RC Twoports for Single-Amplifier Biquad(Cont.)

– Pole and zero generation

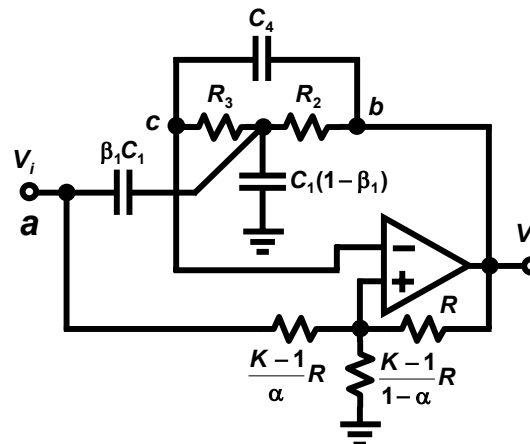


– Application example 1: $\frac{V_o}{V_i} = -\frac{\alpha}{K-1} \frac{1 - \frac{K}{\alpha} T_{cg}}{1 - \frac{K}{K-1} T_{cb}}$ (using RC bridged T)

(i) ENF case A



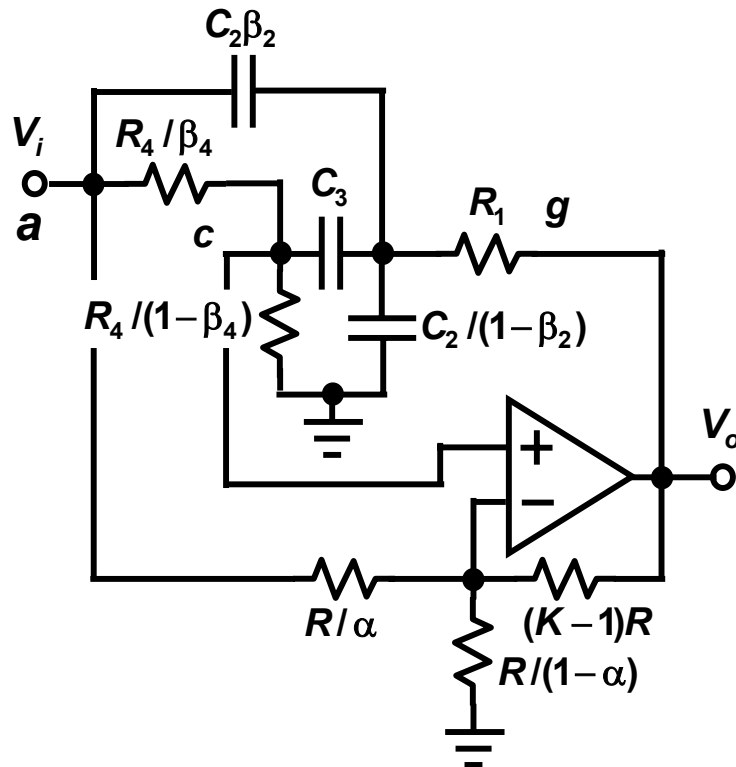
(ii) ENF case B



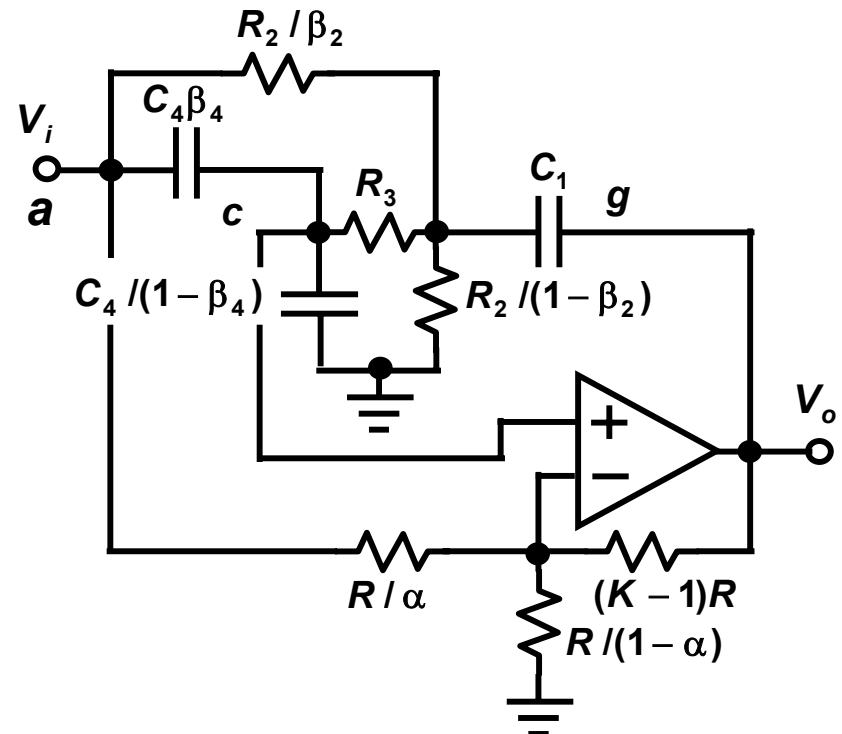
Passive RC Twoports for Single-Amplifier Biquad(Cont.)

- Application example 2: (suitable for EPF) $\frac{V_o}{V_i} = -K \frac{\alpha \frac{K-1}{K} - T_{cg}}{1 - KT_{cg}}$ (using RC ladder, complementary to RC bridged T)

(i) EPF case A

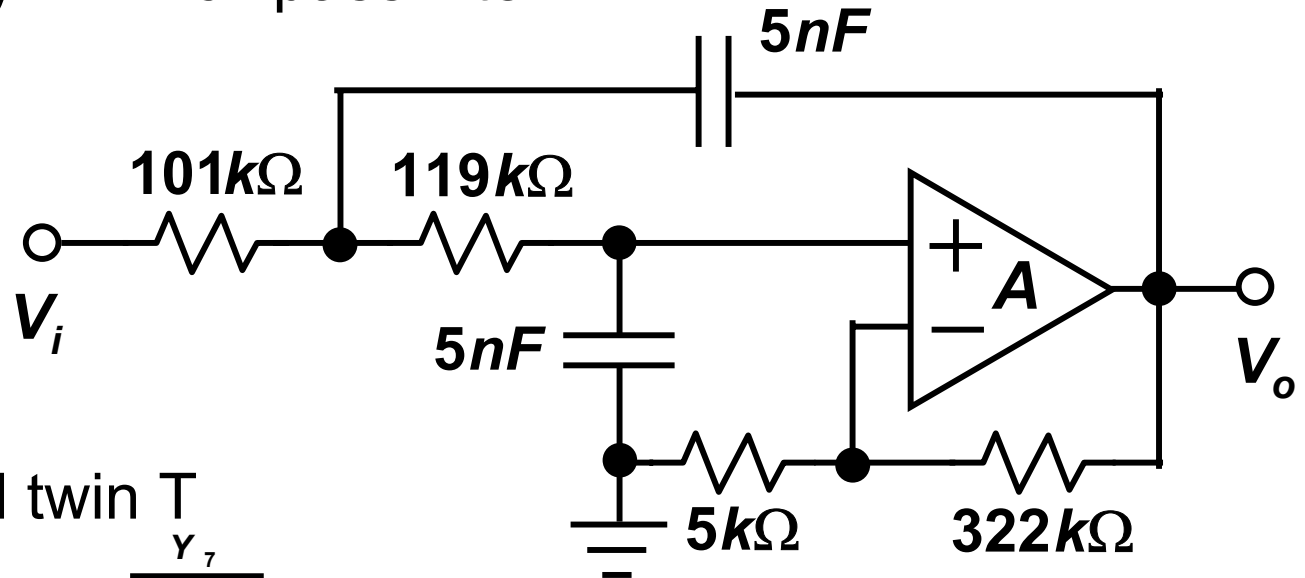


(ii) EPF case B

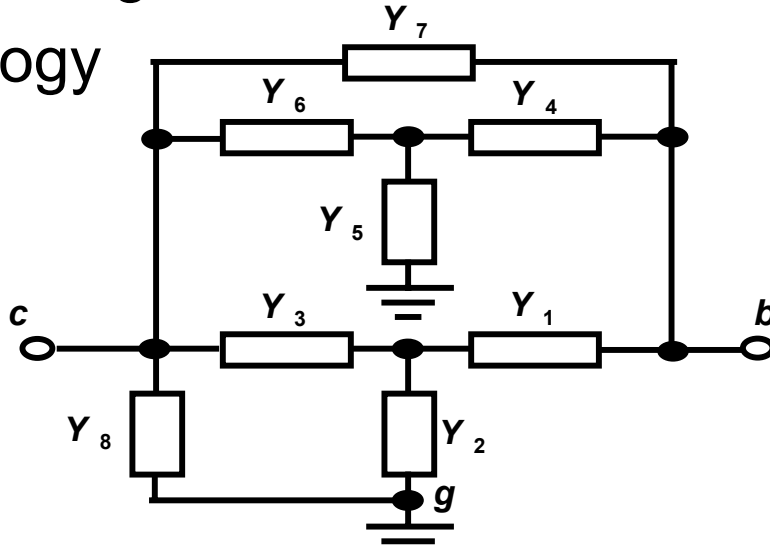


Passive RC Twoports for Single-Amplifier Biquad (Cont.)

- Sallen & Key EPF lowpass filter

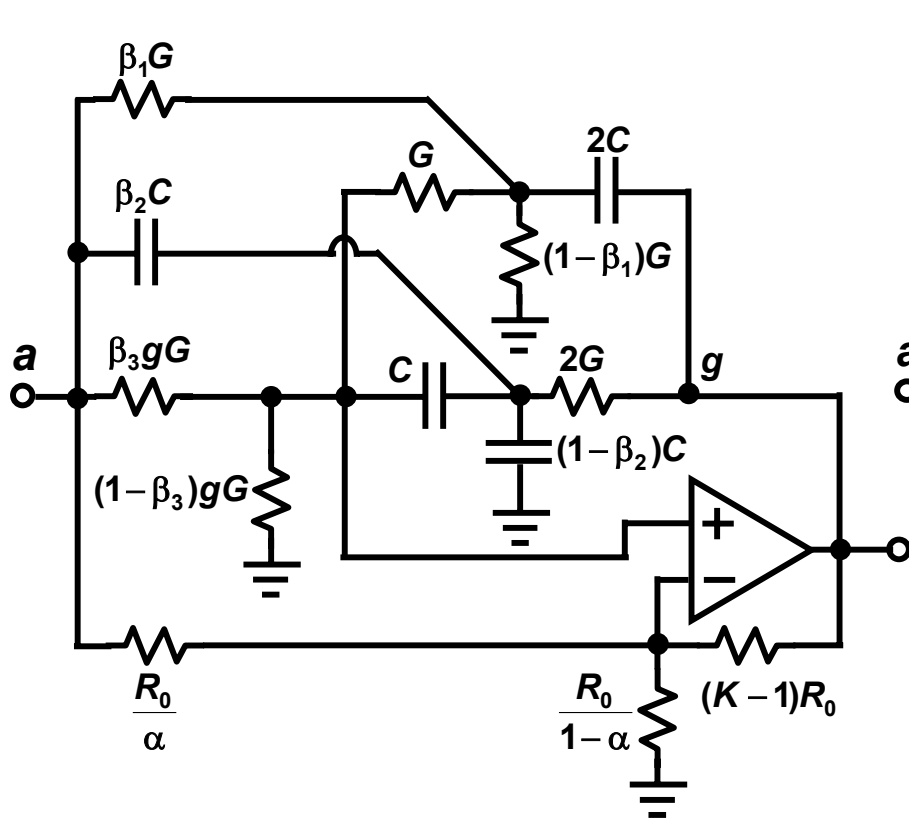


- Loaded bridged twin T
– topology

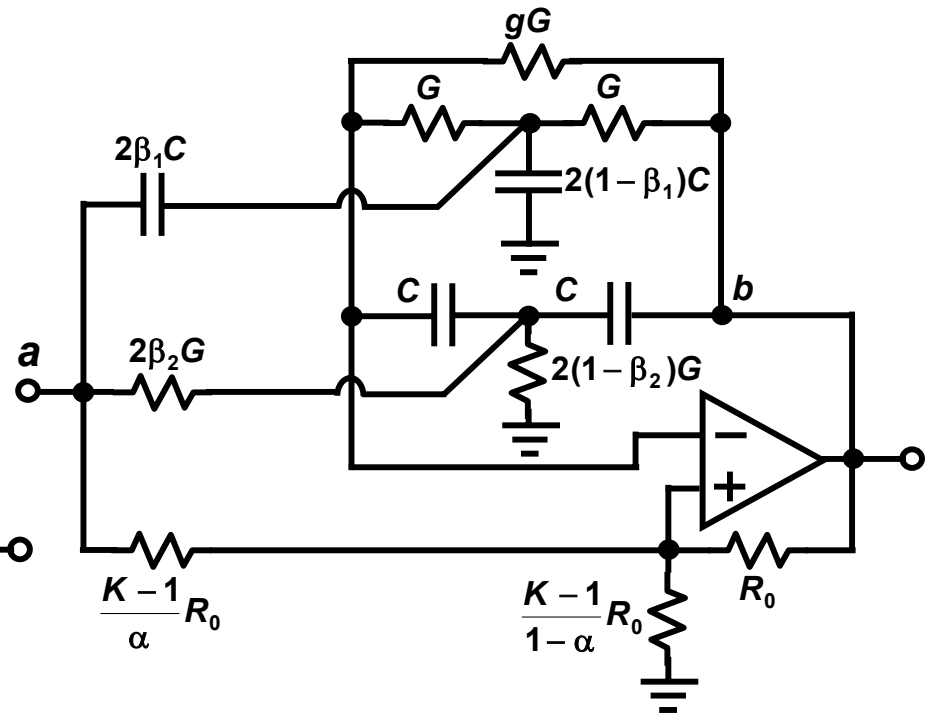


Passive RC Twoports for Single-Amplifier Biquad(Cont.)

– ENF example



– EPF example



Multiamplifier Biquads

- Biquads using a composite amplifier

$\frac{\Delta\omega_0}{\omega_0}$ and $\frac{\Delta Q_0}{Q_0}$ are proportional to $\left|\frac{1}{A(j\omega_0)}\right|^2$, whereas for the SAB they were proportional to $\left|\frac{1}{A(j\omega_0)}\right|$.

- Biquads based on general impedance converter (GIC)

Biquads based on Antoniou's general impedance converter has the best sensitivity performance among the many proposed two-amplifier circuits.

Multiamplifier Biquads

- Biquads using three OPAMPs
 - state-variable realization first proposed by Kerwin, Huelsman, and Newcomb (KHN)
 - greater versatility
 - (1) different types of transfer functions to be realized simultaneously at the available OPAMP outputs
 - (2) tuning will be noninteractive because the additional OPAMPs isolate different parts of the circuit from each other.

Multi-amplifier Biquads VS. SABs

- Single-amplifier biquads (SABs)
2nd-order RC network together with a single amplifier can be used to realize an arbitrary 2nd-order transfer function
- Multi-amplifier circuits
 - Advantages
 - (1) simultaneous availability of different transfer functions at different outputs
 - (2) the ease of tuning and noninteractive adjusting of filter parameters, because the design equations are seen to be decoupled by the insertion of additional OPAMPs

Multiplier Biquads VS. SABs (Cont.)

- Disadvantage
 - increased expense of additional OPAMPs and power consumption
 - Sensitivity is not significantly better than well-designed SABs
- If low sensitivity, in particular to the active device tolerances, is of major concern, composite amplifier may be preferable. SABs based on composite amplifier has small w_0 and Q errors which is ideally proportional to $\left| \frac{1}{A(jw_0)} \right|^2$ instead of $\left| \frac{1}{A(jw_0)} \right|$ of the general one OPAMP approach